A Model of Gross Capital Flows: Risk sharing and Financial Frictions

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October 2022

Abstract

This paper builds a two-country model of gross capital flows in which agents share tradable output risk using two bonds, subject to stochastic collateral constraints. Equilibrium portfolios are short in domestic bonds and long in foreign bonds because the endogenous movements of the real exchange rate provide a hedge against domestic output shocks. Under negative domestic shocks, these external positions transfer wealth from home to abroad. In an application to the Great Recession, the model shows that such wealth transfer from the US mitigated the consumption drop abroad. Quantitatively, financial frictions can account for at least 55% of the collapse in gross flows of the US in 2008.

JEL Classification: F37, F41, E44

Keywords: Capital flows, Gross flows, Risk sharing, Financial frictions

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Cross-border financial transactions have been growing steadily since the early 1980s, and then experienced a dramatic collapse in 2008. Gross flows, defined as the changes in international investment positions in both assets and liabilities due to transactions, increased from 2% of GDP in 1970 to 26% in 2007 in the United States fueled by financial development and reduced capital controls. Subsequently, the gross flows dropped by 3.6 trillion dollars[1] during the Great Recession, attaining the lowest level since 1980. The sudden collapse of gross flows during the financial turmoil opened a debate about whether gross flows matter, especially for developed countries, and about their implications for the real economy.

Classic studies of integrated financial markets have mainly focused on net flows, defined as the changes in international liability positions net of asset positions, or current accounts.[2] Between 2007 and 2009, net flows fell by 3% of GDP in the United States, compared to a 23% drop in gross flows over the same period. This contrasts with the experience of emerging markets, which tend to have more volatile net flows and less volatile gross flows relative to developed countries (Broner et al. 2013). Motivated by these observations, I develop a theory of gross capital flows driven by two factors, economic fundamentals and financial frictions, and evaluate their quantitative importance for integrated financial markets.

Accounting for gross flows is important because they affect the size of wealth transfers across countries, which are ultimately at the heart of consumption smoothing. One motivating observation during the Great Recession is that the sudden collapse of gross flows coincided with a sharp appreciation of the dollar and severe financial frictions. Because the United States mostly borrows in the dollar and invests in foreign currencies (Gourinchas and Rey 2007a), a collapse in both inflows and outflows, combined with a simultaneous appreciation of the dollar, results in a limited wealth transfer from the United States to the Rest of the World. A natural question that arises is, how would the welfare of each economy change compared to a classic environment of only net flows? Also, how much of the collapse in gross flows was driven by financial frictions as opposed to economic fundamentals?

The main hypothesis is that households have a motive for sharing tradable output risk which leads them to hold international investment positions, subject to collateral constraints. If domestic and foreign tradable goods are complements, it is optimal for households to im-

[1] equivalent to a 25 percentage point drop in gross flows to GDP, from 26% in 2007 to 1% in 2008.

[2] Current accounts are defined as Exports of goods and services and income receipts net of Imports of goods and services and income payments. In the Balance of Payments, the sum of current account and capital account (which is usually small) balances with net flows due to the accounting identity in theory, but not in practice (Obstfeld, 2012).

port more foreign goods when domestic output is hit by a positive shock, and vice versa in response to a negative shock. Unless shocks to tradable output are perfectly correlated across countries, all households are better off by sharing output risk internationally via goods trading. In such cases, cross-border financial transactions are the instrument with which the risk sharing is implemented. The degree of risk sharing is bound by the value of the collateral.

To examine the mechanism of risk sharing through international financial transactions under financial frictions, this paper builds an open economy model of portfolio choice with two symmetric countries and two real bonds, where short positions are subject to stochastic collateral constraints. Each bond is denominated in the aggregate price index of the given country and promises to pay one unit of the aggregate consumption good next period. Each country produces one tradable good, whose output is exogenous and follows a stochastic process. In the presence of home bias in the aggregate consumption basket, real exchange rates (RER) are determined by relative prices of home to foreign goods. In this model, the RER plays a key role in the decision of investment positions for households, since it sets the returns on foreign bonds relative to domestic ones.

The equilibrium portfolio takes a short position (households borrow) in domestic bonds and a long position (households save) in foreign bonds. This is because the endogenous movements of the real exchange rate make this portfolio optimal for risk sharing. For example, when a positive shock hits tradable output in one country, its domestic tradable goods become less valuable relative to the foreign tradable goods, which implies real exchange rate depreciation. Combined with liabilities in domestic bonds and assets in foreign bonds, such depreciation of the RER results in an increase in the value of net international investment position (NIIP) for households. This increase in NIIP in turn can be used to purchase more foreign goods, which are complements to domestic goods in the calibrated model.

Adjustments to the equilibrium portfolio take place under two circumstances. First, when the output of either country is hit by a shock, gross flows arise as a difference in gross positions. Negative shocks to output in any country result in lower positions for both domestic and foreign bonds because there is less risk to insure against. This implies that the model predicts a positive correlation of gross flows and output, just as observed in the data. Secondly, if the collateral constraints tighten, gross positions decrease and gross flows become negative. In the model, these two channels jointly drive the movements of gross flows.

To assess the quantitative power of these channels, I calibrate the model to data of the

United States, which experienced a dramatic collapse in gross flows during the Great Recession. Using the data on gross output for years 2000-2019, I estimate the parameters of stochastic processes in the model and generate a number of testable implications regarding gross flows. Specifically, I feed in the paths of realized output in the US and the Rest of the World (RoW) and show that the model replicates the key stylized facts regarding gross flows. First, the simulated gross flows match the observed data closely, not only in terms of their magnitude and volatility, but also the patterns of gradual boom and sudden collapse. In addition, the model asset and liability flows are highly correlated and pro-cyclical, which is one of the key characteristics of gross flows in developed countries. Moreover, risk sharing and financial frictions can account for nearly all of the volatility in gross flows. In particular, the simulated results generate the collapse during the 2008 crisis to the same extent as in the data, while the financial friction channel alone can explain 55% of the decline. Importantly, this prediction is accompanied by a steep appreciation of the dollar, resulting in a reduced net external position of the US relative to RoW. This is in line with the exchange rate valuation effects documented by Lane and Shambaugh (2010) and especially Bénétrix et al. (2015) during the financial crisis.

The main result is that the observed fluctuations in gross flows mitigated a part of the consumption drop in the aftermath of the Great Recession in RoW, due to significant transfers of wealth from the US. Through the risk sharing mechanism of gross flows, the RoW consumption dropped by 3.3% from 2008 to 2009, which would have been 3.35% under a classic single-bond economy with only net flows. These gains in consumption for RoW came at the cost of losses in consumption in the US, which instead enjoyed a higher level of consumption in 2007. Overall welfare in the world was improved through the risk sharing channel during the sample period. It is noteworthy that the magnitudes and directions of welfare gains depend on the levels of Armington elasticity. For example, a lower level enhances the welfare gains of RoW during the financial crisis.

Literature review There is a growing literature that models gross flows, or more generally inflows and outflows separately. Tille and Van Wincoop (2010) build a model of gross flows in a two-country setting with productivity shocks and develop a first- and second-order approximation method. I take a step forward by adding financial frictions and solving the model globally, and hence analyzing gross flows during the financial crisis. Dou and Verdelhan (2015) and Gourio et al. (2015) focus on volatility and uncertainty shocks as an amplifier of fluctuations in gross flows, but they do not explain the gradual increase of gross flows. My paper models both crisis shocks and deterministic trends on collateral constraints, which incorporate not only large fluctuations during the financial crisis but also a steady growth of gross flows in the run-up to the crisis. Davis and Van Wincoop (2018) show that globalization leads to an increase in inflows and outflows, focusing on long-run trends and using a linear approximation method. I complement their approach by studying fluctuations of inflows and outflows in response to extreme shocks during the financial crisis using the global method. Recently, Caballero and Simsek (2020) propose a mechanism where gross flows are in part generated by a model assumption because investors are required to sell their foreign assets in the face of liquidity shocks. In comparison, my paper endogenously generates gross flows without assuming any intrinsic differences between domestic and foreign assets except for their currency denominations and hence returns. Finally, Kumhof et al. (2020) model gross flows that are driven by the liquidity demand and emphasize the role of cross-border financial intermediaries, using a local approximation for the solution method. My paper is also motivated by the role of financial intermediaries, which is the target of financial frictions calibration. The departure point of my paper is that I focus on the risk sharing motive as a driver of gross flows rather than the liquidity demand, and I provide a global solution in order to study the financial crisis event.

The two main channels in the model, risk sharing and financial frictions, are based on extensive literature. For example, papers in the international diversification literature traditionally focus on the long-run external positions (Stockman and Dellas 1989, Baxter et al. 1998, Heathcote and Perri 2013, among others). The departure point of this paper is to study the dynamics of international risk sharing over time, compared to a long-run international diversification that is not time varying. The literature of macro-finance with various financial frictions, and in particular collateral constraints, has seen an explosive growth especially since the financial crisis (Kiyotaki and Moore 1997, Bernanke et al. 1999, Gertler and Kiyotaki 2010). This paper shows that occasionally binding collateral constraints play a key role in gross flows, both in their gradual growth and a sudden collapse.

Empirically, there are some of the well-known characteristics of gross flows documented in the literature. For example, Broner et al. (2013), Rey (2015), and Forbes and Warnock (2012) document large and volatile movements of gross flows, using an extensive panel of countries. More recently, Avdjiev et al. (2022) and Davis et al. (2021) analyze different types of gross flows and the contribution of global factors. In this paper, model-generated gross flows are broadly consistent with these key characteristics, such as a sudden collapse during the financial crisis and a high correlation between inflows and outflows.

Finally, this paper contributes to the international macroeconomics literature by solving a model of gross capital flows using a global method. This contrasts with the existing papers on international capital flows which have relied on linear approximation approaches. The global solution method is necessary to study extreme events such as the Great Recession, and the effects of financial frictions through occasionally binding collateral constraints. The non-linear solution technique is based on the time iteration algorithm proposed by Kubler and Schmedders (2003). One closely related study which adopts the global solution method to international portfolio choice is Stepanchuk and Tsyrennikov (2015). Their focus is on net flows and net foreign asset positions, however, while this paper develops a model of gross flows.

The rest of this paper is organized as follows. In Section 1, key characteristics of gross flows along with international investment positions are described. Section 3 studies a simple environment of complete markets with analytic solutions. Section 2 lays out the model environment. Section 4 brings the model to the data. Conclusion follows.

1 Data

In this section, the concept of gross flows and their key characteristics are described. The data observations on gross flows and characterizing statistics motivate the model design in the following section (Section 2). These characteristics are revisited in the Quantitative Analysis (Section 4), comparing the model results to the data.

1.1 Overview of gross flows

Concept of gross flows Gross capital flows are changes in international investment positions (IIP) due to transactions. IIP is a balance sheet of a country that records both *assets*, which are the financial claims on nonresidents (cross-border investments by domestic residents), and *liabilities*, which are the claims by nonresidents on residents (cross-border borrowings by domestic residents). *Asset flows* are changes in asset positions due to net acquisitions of foreign financial assets by domestic residents. Analogously, *liability flows* are changes in liability positions, which are equivalent to the net incurrence of liabilities by domestic residents. Gross capital flows are defined as the sum of asset and liability flows.

Boom, collapse, and slow recovery Focusing on the sample period of 2000-2019 in the United States, I document some key characteristics of gross flows. First, asset and liability flows are highly correlated with the correlation coefficient of 0.94, as the left panel of Figure 1 demonstrates. This observation is consistent with the empirical literature, where Broner et al.

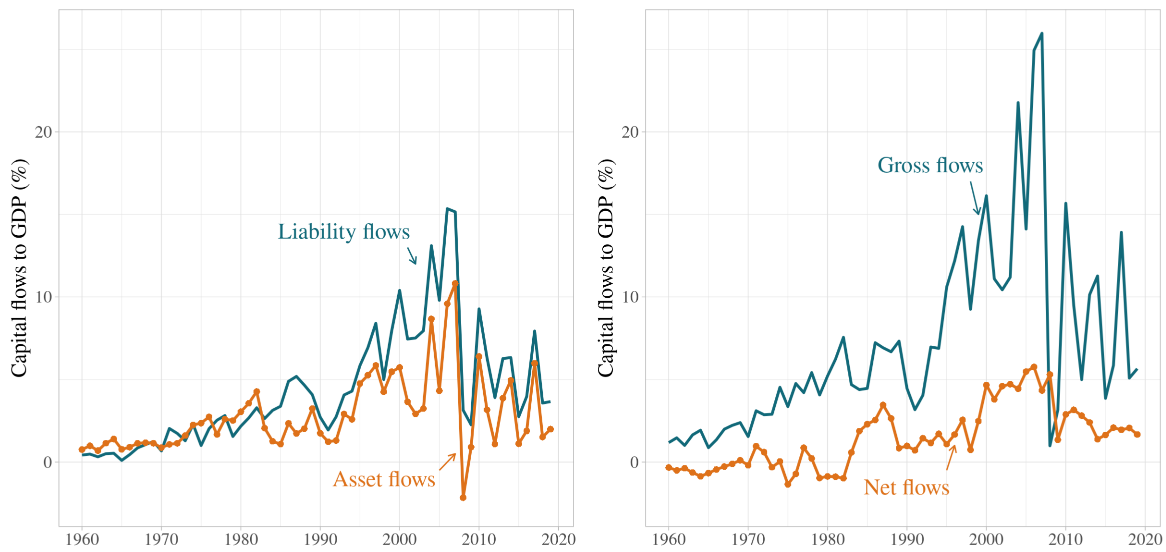

Figure 1: Capital flows of the United States against the Rest of the World

Source: Bureau of Economic Analysis and author's calculations

(2013) document that the correlations have been increasing over time across 103 countries, especially after 2000. Davis and Van Wincoop (2018) also document increasing correlations over the sample of 128 countries, and suggest that it is a result of increased financial and trade globalization. In a newly constructed dataset, Avdjiev et al. (2022) show that the high correlation between asset and liability flows is driven by banking flows, which will be elaborated further in the next paragraph. Second, gross flows are larger in levels, and much more volatile than net flows. As the right panel of Figure 1 depicts, gross flows peak 26% of GDP in 2007, and experience a sharp collapse to 1% of GDP in 2008. On the other hand, net flows, which are defined as liability flows net of asset flows, move from 4% to 5% during the same period, or 6% (2006) to 1% (2009) from peak to trough. While the net flows also have seen a significant change during the recession, the gross flows to GDP (7% std) are much more volatile than the net flows to GDP (1.5% std).

1.2 Structure of gross flows and positions

Structure of gross flows Both asset and liability flows are further decomposed into *debt* and *equity* flows (Lane and Milesi-Ferretti 2007 and Avdjiev et al. 2022), as Figure 2 summarizes in a diagram. Debt flows are composed of Portfolio investments in debt securities, Other investments, which are mostly banking flows, and Direct investment debt. On the other hand, Direct investment equity and Portfolio investments in Equity and investment

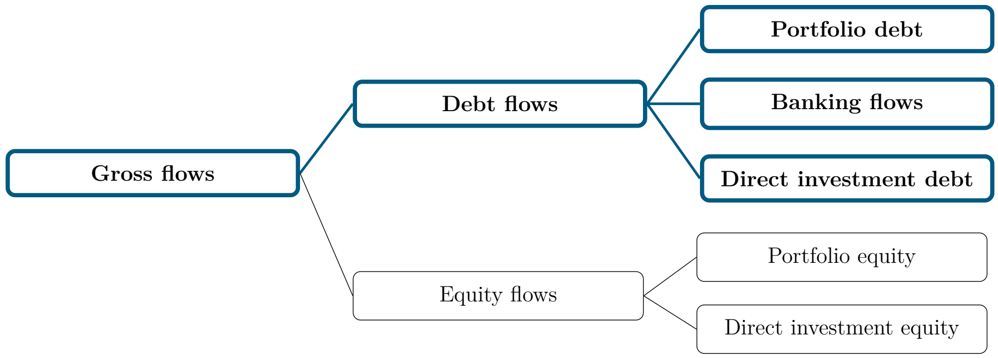

Figure 2: Decomposition of gross flows

fund shares constitute equity flows.[3] Comparing the magnitudes of *debt* and *equity* flows in the US from 2000 to 2019, debt gross flows are on average 76% of the total gross flows in absolute values.[4] Among the debt flows, banking flows account for a majority of them, as recently studied by Avdjiev et al. (2022).

Motivated by these observations, the model in the next section abstracts from cross-border equity transactions. Instead, it casts the dynamics of gross flows into a tractable model with internationally traded bonds subject to collateral constraints. In the following quantitative section, the entire capital flows from the data, including both debt and equity flows, are compared with the model results. Also, the sizable role of the banking sector in the gross flows is reflected in the setup of collateral constraints, following a long literature of macro finance. This is in line with a recent study by Kumhof et al. (2020) that emphasizes the role of banking in driving international capital flows.

Structure of gross positions and currency composition Analogous classifications can be applied to external asset and liability positions, and different types may prevail on a country's external balance sheet. In particular, the US external positions are characterized as mostly *debt* on the liability side (i.e., portfolio debt, banking liabilities, and direct investment debt), and mostly *equity* on the asset side (i.e., portfolio equity and direct investment equity). Such characteristics of external positions drive the currency composition and key

[3] Financial derivatives, whose data is recorded in net positions only, and reserve assets, whose amount is small in the US, are not included in the classifications.

[4] In deriving the average fraction of debt flows, I calculate mean(abs(debt gross flows)/abs(total gross flows)) = 0.76 because capital flows can take negative values. Data source is Bureau of Economic Analysis, U.S. International Transactions, Expanded Detail, sample period 2000-2019, annual. The fact that debt flows are a lion's share of gross flows owes to the part that liability flows are larger than asset flows, and most of liability flows are debt flows. Also, there are some years where the debt gross flows are large in negative values and total gross flows are small in positive values, for example in 2008, and vice versa. Separate illustrations for asset and liability flows can be found in Appendix Figure 15.

features of gross flows that motivate the model design.

More specifically, it is widely documented that the US mostly borrows in safe dollar debt and invests in risky equity abroad, acting as a "world's venture capitalist" (Gourinchas and Rey 2007a, Lane and Milesi-Ferretti 2007). Safe dollar debt includes both banking and other types of debt securities, and banking sector accounts for a majority of external debt liabilities in the US and other advanced economies (Avdjiev et al. 2022). On the asset side, equity positions abroad are usually assumed as investments in local currencies, using the geography of equity issuance as a predictor for the currency denomination. Based on this assumption, Benetrix et al. (2019) estimate that the US holds 85% of external liabilities in dollars and 63% of external assets in foreign currency on average from 2000 to 2017.[5] Such currency composition contrasts with emerging market and developing economies, where countries tend to borrow in foreign currencies compared to advanced economies (Bénétrix et al., 2015).

In the following quantitative exercises (section 4), in line with above currency composition, US households in the model borrow in domestic bonds (denominated in domestic currency) and invest in foreign bonds (foreign currency) in equilibrium. It should be noted that as the model abstracts from cross-border equity holdings, these currency compositions are captured entirely by bond positions. While the model does not feature the fluctuations in equity values, it nevertheless explains currency valuation effects in the external assets. Moreover, as the Great Recession exercise shows in the following quantitative section, the value of foreign bonds drops during the crisis in the model. This is in line with the sharp drop of foreign equity value in the data during the same period.[6]

1.3 Real exchange rates and gross flows

One of the key links highlighted in this paper is that external portfolios and hence gross flows are related to endogenous movements of real exchange rates. This relationship between gross flows and real exchange rates was especially pronounced during the Great Recession in the United States. In Figure 3, a time series of real exchange rate of the US dollar is plotted against GDP and gross flows to GDP ratio.

[5]It is noteworthy that on the asset side, there is a good portion of dollar assets, which are mostly debt. This can be considered as risky dollar debt, such as sovereign, corporate, or banking debt abroad.

[6]More technically, value of external assets denominated in foreign currency can be affected by two sources: value of assets in local currency and exchange rates. In the data, both sources prevail especially for the part of foreign equity assets. During the financial crisis, for example, equity prices drop sharply and foreign currencies against US dollars depreciate simultaneously. In the model, only the part of foreign currency depreciation is captured, which moves in the same direction of local currency equity value during the times of crisis.

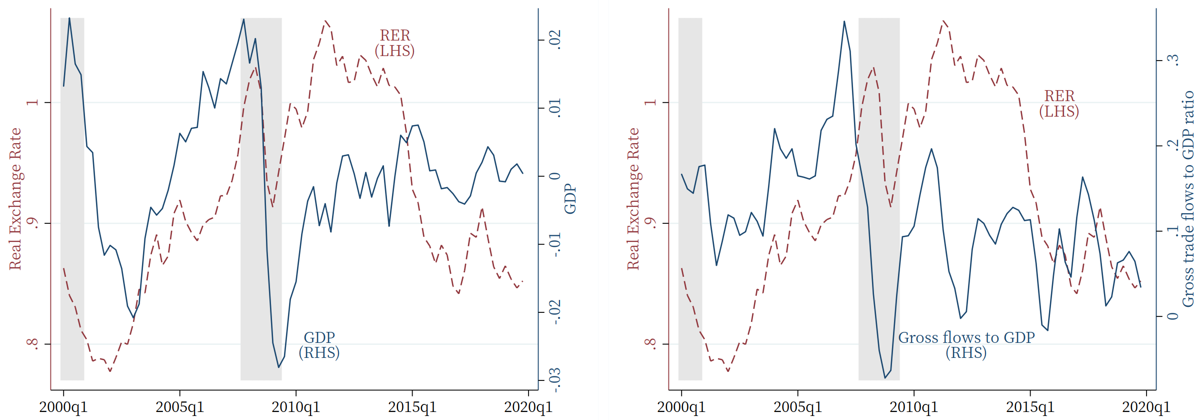

Figure 3: Real exchange rate, GDP, and gross capital flows

Note: GDP series on the left panel are de-trended using HP filters. Gross capital flows to GDP ratio is a 3-quarter moving average. Source of real exchange rate is BIS, broad based effective exchange rate indices, real, quarterly data. GDP and gross flows series are sourced from BEA. A decrease in RER series indicates an appreciation of USD.

On the left panel of Figure 3, a sharp appreciation of the US dollars during the Global Financial Crisis is depicted. Combined with a external currency composition of the US, which mostly borrows in dollars and invests in foreign currencies, this appreciation of the USD during the crisis resulted in a significant wealth transfer from the US to the rest of the world (see Bénétrix et al. (2015) for a comprehensive analysis of external currency exposures). Such wealth transfer is also known as "exorbitant duty" by Gourinchas and Rey (2022), in contrast to the "exorbitant privilege" that the US has enjoyed as the reserve currency issuer. This paper highlights that these (endogenous) movements of exchange rates are at the heart of external asset and liability holdings, and proposes a framework featuring the observed appreciation during the crisis and currency composition of external portfolio.

A sharp appreciation of real exchange rates during the crisis also coincides with a collapse of gross flows, as the right panel of Figure 3 shows. A positive correlation between real exchange rates and gross flows to GDP ratio in the United States can be observed in this figure, as gross flows increase during the times of USD depreciation and experience a significant reversal during the event of appreciation. In the following model section, the correlation of gross flows and real exchange rates is explained as an equilibrium outcome.

It should be noted that the above relationship among real exchange rates, output, and gross flows are particular for cases of developed countries and the US as the reserve currency country. It is well known that emerging market currencies tend to depreciate when output growth is low (Frankel and Rose 1996). In the following, the model (Section 2) is set up so that it is applicable for both advanced and developing countries. In the Quantitative Exercise section (Section 4), the model is calibrated targeting the US, and the results are suitable only for the developed economies. It is possible, however, to apply the model to developing countries after adjusting some of the key parameter values, which is discussed more in the Sensitivity Analysis section (Section 4.4).

1.4 Risk sharing and gross flows

As a final piece of empirical evidence, in this section I describe how gross flows are positively correlated with a measured degree of international risk sharing. The main hypothesis in this paper is that households hold international investment positions in order to share tradable output risk across countries. Then, do we observe any relationship between the degree of risk sharing and gross capital flows? In particular, if the hypothesis is correct, the degree of international risk sharing would be higher when gross flows increase.

In order to show the relationship between risk sharing and gross flows, I measure the degree of international risk sharing by running a regression of cross-country output growth on consumption growth using a panel of advanced countries (Bai and Zhang, 2012).[7] Figure 13 in Appendix shows that the time series of measured risk sharing and amounts of gross flows in advanced economies move in tandem, where gross flows lead the cycle. In particular, after an explosive growth of international capital flows since the early 2000's, the level of cross-border risk sharing among advanced economies increased significantly in the following years. Also, since the collapse of gross flows during the Great Recession and a slow recovery afterwards, the level of risk sharing is also trending down until recently. Explained in Appendix with more details on the data and measurement methods, this evidence suggests that there is a strong positive correlation between risk sharing and gross flows observed in the data. Motivated by this observation, I build a model where the risk sharing motive is one of the main drivers that generate gross flows in the next section.

[7]The key idea is that under perfect risk sharing, idiosyncratic shocks on output across countries would not have any correlation with cross-country consumption growth. Therefore, if there is a non-zero correlation between output and consumption across countries within a time period, it indicates imperfect risk sharing.

2 The Model

In this section, I describe the physical environment of the benchmark model and financial market structure. Following international portfolio choice models such as Baxter et al. (1998) and Tille and Van Wincoop (2010), I model an exchange economy where each country owns a Lucas Tree (Lucas, 1978) that produces a tradable good following stochastic processes. This section focuses on the core mechanism of gross capital flows driven by two channels, risk sharing and financial frictions.

Physical environment In each period of time $t = 0, 1, \ldots \infty$, an exogenous state denoted as $s_t \in S$ realizes. I denote the history of states from time 0 to time t as $s^t = (s_0, s_1, \ldots s_t)$, which is also called as a node in the event tree. The root of the event tree is given as s_0. The probability of a node s^t realization is denoted as $\pi(s^t)$ in terms of time-0 probability, and the chance of node s^{t+1} realization given the history s^t is denoted as $\pi(s^{t+1}|s^t)$. Events follow a Markov process, which is specified in the following paragraph.

In the model, there are two countries, 1 and 2, which are populated by a continuum of identical households with measure 1 in each country. Throughout this section, I focus on the problems of country 1 since the settings are symmetric across countries. There are two tradable goods $\boldsymbol{y} = (y_1, y_2)$ in the world, whose outputs are given as an exogenous process. Based on Lucas (1978), I assume that each country $i = 1, 2$ owns a Lucas Tree that produces tradable good output y_i. The world output $\boldsymbol{y}(s^t)$ follows a log normal AR1 process, with a mean of $\boldsymbol{\rho} \log \boldsymbol{y}(s^{t-1})$ and covariance of Σ. To summarize,

$$\begin{bmatrix} \log y_1(s^t) \\ \log y_2(s^t) \end{bmatrix} = \boldsymbol{\rho} \begin{bmatrix} \log y_1(s^{t-1}) \\ \log y_2(s^{t-1}) \end{bmatrix} + \boldsymbol{\varepsilon}(s^t). \tag{1}$$

where $\boldsymbol{\varepsilon}(s^t) \sim N(0, \Sigma)$. Notice that I abstract away from the existence of non-tradable goods in this paper and follow a classic literature of international real business cycle models and portfolio choice with only tradable goods (Backus et al., 1992, Heathcote and Perri, 2013).

Household utility Each household is risk averse and demands a basket of two tradable goods. Utility functions are assumed to be symmetric across countries. Flow utility has a constant relative risk aversion γ with respect to the aggregate consumption basket c^1. Final consumption good is aggregated with a CES technology where elasticity of substitution is σ, a weight on home good is ω_H (home bias), and a weight on foreign good is given as

$\omega_F = 1 - \omega_H$ as in the complete markets model (section 3).

$$u(c^1(s^t)) = \frac{c^1(s^t)^{1-\gamma}}{1-\gamma},\ c^1(s^t) = \left(\omega_H^{\frac{1}{\sigma}}\left(c_1^1(s^t)\right)^{\frac{\sigma-1}{\sigma}} + \omega_F^{\frac{1}{\sigma}}\left(c_2^1(s^t)\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where c_i^j denotes the consumption of good i by consumers in country j.

Household budget constraint Households decide on the amount of consumption for two tradable goods (c_1^1, c_2^1), and financial portfolio comprised of domestic bonds (a_1^1), foreign bonds (a_2^1), and domestic equity (θ^1). Their income is generated by returns on financial assets, including dividend payments on equity holdings. Equity holdings $\theta^1 \in [0,1]$ give a shareholder claims to a fraction of output y_1, which is a dividend to the Lucas Tree in country 1. Bond i promises a unit of *final* good in country i in the next period, regardless of the state. To summarize, the budget constraint of household in country 1 is given as the following.

$$\begin{aligned}\sum_{i=1}^{2} p_i^1(s^t)c_i^1(s^t) + q_1(s^t)a_1^1(s^t) + e(s^t)q_2(s^t)a_2^1(s^t) + p_1^1(s^t)Q_1(s^t)\theta^1(s^t)\\ \leq a_1^1(s^{t-1}) + e(s^t)a_2^1(s^{t-1}) + p_1^1(s^t)\left(y_1(s^t) + Q_1(s^t)\right)\theta^1(s^{t-1})\end{aligned} \tag{3}$$

where p_i^1 is relative price of tradable good i to the final good in country 1, q_i is the price of bond i, e is the exchange rate (price of country 2 final good in terms of country 1 final good), and Q_i is the price of equity i. The price of final goods in country 1 is used as a numeraire.

Stochastic collateral constraints It is assumed that the bond purchases are subject to stochastic collateral constraints. The maximum amount of total borrowing by country j households cannot exceed a fraction $\chi(s^t)$ of total equity value $p_j^j(s^t)Q_j(s^t)\theta^j(s^{t-1})$. Formally, for the household in country 1, collateral constraint for the total borrowing is:

$$a_1^1(s^t)\mathbb{1}_{\left(a_1^1(s^t)<0\right)} + e(s^t)a_2^1(s^t)\mathbb{1}_{\left(a_2^1(s^t)<0\right)} \geq -\chi(s^t)p_1^1(s^t)Q_1(s^t)\theta^1(s^{t-1}) \tag{4}$$

Here, the fractions of collateralization $\chi_i(s^t)$ are stochastic, following a Markov process. In particular, there can be a period of *crisis*, where collateral constraints in both countries switch to lower levels than the *normal* times, and with a certain probability the constraints resume to normal levels.[8] If the collateral constraints bind during the crisis time, then households in both countries experience elevated financial frictions and more incomplete international

[8]Similar settings of credit shocks can be found in the literature related to the Great Recession, such as in Khan and Thomas (2013), and the global nature of credit shock is in line with the Global Financial Cycle (Miranda-Agrippino and Rey 2021).

financial markets.

This setting is under the assumption that in international financial markets, borrowers can default on bonds and the liquidation value of the Tree as a collateral can be zero with a probability $(1-\chi^1(s^t))$, following Jermann and Quadrini (2012).[9] Notice that the constraint is imposed on *gross* liabilities, rather than on *net* liabilities. This setting is motivated by empirical observations that a majority of gross flows originate from debt flows, and in particular banking flows as discussed in Data section 1. As studied by Bruno and Shin (2015), banking flows follow leverage cycles, which is closely related to the size of balance sheets. In other words, collateral constraints on gross flows can be interpreted as financial constraints faced by banks on their balance sheets, such as capital requirement regulations, that are embedded in the households' collateral constraints for a two-country two-bonds model.

Traditionally in the open economy literature, financial frictions play an important role on capital flows in developing economies.[10] However, especially after the financial crisis, a large body of macro-finance literature shows that financial frictions are an integral part of macroeconomic fluctuations in advanced economy as well.[11] This paper extends the collateral constraints used to study the domestic macroeconomy in advanced markets into the international financial flows.

Household's problem Each household maximizes her expected utility at time 0, which is $\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)u(c_1(s^t))$, under the budget constraint (equation 3) and borrowing constraint (equation 4).

Market clearing Goods markets clear for each state, and bonds have zero net supply. Equities are owned by domestic households only.

$$\sum_{j=1}^{2} c_i^j(s^t) = y_i(s^t),\ \ \sum_{j=1}^{2} a_i^j(s^t) = 0,\ \ \theta^i(s^t) = 1,\ \forall s^t \in S^t,\ i = 1,2 \tag{5}$$

[9] More details of the microfoundation behind the collateral constraint can be found in Appendix B.

[10] For example, Bianchi et al. (2018) study the role of financial frictions in the optimal choice of international reserves and long-term debt in emerging markets. Antràs and Caballero (2009) emphasize how heterogeneity in financial development can result in trade and financial flows to be complements in less financially developed economies. Davis et al. (2021) find that in countries with higher net debt liabilities, the global financial cycle factor has stronger effects on both gross and net flows.

[11] There is a large body of literature studying the interaction between financial frictions and macroeconomics, with or without explicit modeling of financial intermediaries, especially after the Global Financial Crisis in advanced economies such as the United States (A comprehensive survey of macroeconomic models with financial frictions can be found in Brunnermeier et al. (2012), including seminal papers such as Bernanke et al. 1999 and Gertler and Kiyotaki 2010.)

Net wealth fraction and recursive formulation In order to solve the model, I transform the household's problem in a recursive form. I first define *wealth fraction* (w), which is the country 1 household's net bond repayment plus dividend payment normalized by the world output. This normalization is designed in a way that in equilibrium, w is equal to the country 1's fraction of cash-in-hand (output plus net bond repayment) normalized by the world output. Formally, net wealth fraction of household $i \in [0,1]$ at the node s^{t+1} is:

$$w^i(s^{t+1}) = \frac{p_1^1(s^{t+1})y_1(s^{t+1})\theta^1(s^t) + a_1^1(s^t) + e(s^{t+1})a_2^1(s^t)}{p_1^1(s^{t+1})y_1(s^{t+1}) + e(s^{t+1})p_2^2(s^{t+1})y_2(s^{t+1})} \tag{6}$$

Notice that portfolio decisions at the end of time period t $(a_i^1(s^t), \theta^1(s^t))$ determines an individual's relative net wealth at the beginning of period $t+1$ $(w^i(s^{t+1}))$, depending on the realization of output and hence equilibrium exchange rate in time $t+1$.

In order to express the household's problem in a recursive way, define *aggregate net wealth* $W(\cdot)$ which is a sum of individual net wealth fraction of households in country 1:

$$W(\cdot) = \int_{i\in[0,1]} w^i(\cdot)di \tag{7}$$

Following Kubler and Schmedders (2003), a sufficient statistic for the endogenous states of both countries is W, based on zero net supply of bonds and identical individuals. In other words, since the sum of net bond positions in country 1 and 2 should be always zero by the market clearing conditions and the net wealth of all individuals within a country is identical, the aggregate net wealth fraction in country 1 becomes a sufficient statistic for the entire economy and all endogenous variables.

Each individual consumer has rational expectations on the evolution of aggregate net wealth fraction. Conditioning on the previous state s^t, a mapping Γ from an aggregate net wealth fraction $W(s^t)$ along with a current exogenous state s_{t+1} to another net wealth fraction at $(s^{t+1} = (s^t, s_{t+1}))$ is given as

$$W(s^{t+1}) = \Gamma\left(W(s^t), s_{t+1}; s^t\right), \ \forall s_{t+1} \in S \tag{8}$$

Notice that consumers form an expectation that maps today's $W(s^t)$ to tomorrow's $W(s^{t+1})$ for any pair of states $(s_t, s_{t+1}) \in S \times S$. In equilibrium, given an aggregate net wealth fraction $W(s^t)$ and a policy function $a_i^1(W(s^t), s^t)$, the following equation should be satisfied for any node s^{t+1}.

$$W(s^{t+1}) = \frac{p_1^1\left(W(s^{t+1}), s^{t+1}\right) y_1(s^{t+1}) + a_1^1\left(W(s^t), s^t\right) + e\left(W(s^{t+1}), s^{t+1}\right) a_2^1\left(W(s^t), s^t\right)}{p_1^1\left(W(s^{t+1}), s^{t+1}\right) y_1(s^{t+1}) + e\left(W(s^{t+1}), s^{t+1}\right) p_2^2\left(W(s^{t+1}), s^{t+1}\right) y_2(s^{t+1})} \tag{9}$$

Notice that the market clearing condition for equities ($\theta^i(s^t) = 1,\ \forall s^t \in S^t$) is used for the aggregate law of motion. Also in equilibrium, individual net wealth fraction is equal to the aggregate net wealth fraction, $w^i(s^{t+1}) = W(s^{t+1}),\ \forall i \in [0,1]$. A formal definition of consumer's problem in a recursive form is as follows.

$$V_1\Big(w(s), \theta^1; W(s), s\Big) = \max_{c_i^1, a_i^{1\prime}, \theta^{1\prime}} u(c_1^1, c_2^1) + \beta \sum_{s'} \pi(s'|s) V_1\left(w(s'); W(s'), s'\right) \tag{10}$$

$$s.t.\quad \sum_{i=1}^{2} p_i^1\left(W(s), s\right) c_i^1 + q_1\left(W(s), s\right) a_1^{1\prime} + e\left(W(s), s\right) q_2\left(W(s), s\right) a_2^{1\prime} \tag{11}$$

$$\leq w(s)\Big(p_1^1\left(W(s), s\right) y_1(s) + p_2^1\left(W(s), s\right) y_2(s)\Big) + p_1^1(W(s), s) Q_1(W(s), s)(\theta^1 - \theta^{1\prime})$$

$$a_1^{1\prime} \mathbb{1}_{\left(a_1^{1\prime} < 0\right)} + e\left(W(s), s\right) a_2^{1\prime} \mathbb{1}_{\left(a_2^{1\prime} < 0\right)} \geq -\chi(s) p_1^1\left(W(s), s\right) Q_1\left(W(s), s\right) \theta^1 \tag{12}$$

$$W(s') = \Gamma\left(W(s), s'; s\right),\ \forall s' \in S \tag{13}$$

$$w\left(s'; W(s'), a_i^{1\prime}, \theta^{1\prime}\right) = \frac{p_1^1\left(W(s'), s'\right) y_1(s') \theta^{1\prime} + a_1^{1\prime} + e\left(W(s'), s'\right) a_2^{1\prime}}{p_1^1\left(W(s'), s'\right) y_1(s') + e\left(W(s'), s'\right) p_2^2\left(W(s'), s'\right) y_2(s')} \tag{14}$$

Here, I denote the country's net wealth fraction as W and individual's net wealth fraction as w, and suppress the history of states s^t into the state of today $s \in S$, exploiting the Markov process of shocks. Accordingly, $s' \in S$ denotes the state of the next period and $a_i^{1\prime}$ is defined as the portfolio choice of today for the payments tomorrow. Consumer's problem in country 2 is defined analogously, where the country 2's aggregate net wealth fraction is $1 - W(s)$ due to the zero net supply of bonds. Finally, notice that in equilibrium equity holdings are always 1 ($\theta^1 \equiv 1$), which essentially makes the net wealth fraction $w(\cdot)$ the only endogenous variable.

Recursive competitive equilibrium Competitive recursive equilibrium is a collection of value functions $\{V_i\left(w(s), \theta^1; W(s), s\right)\}_{i=1,2}$, law of motion for the aggregate net wealth fraction $\Gamma\left(W(s), s'; s\right)$, consumption allocation $\{c_i^j\left(w(s); W(s), s\right)\}_{i,j=1,2}$, asset holdings $\{a_i^{j\prime}\Big(w(s); W(s), s\Big), \theta^{j\prime}\Big(w(s); W(s), s\Big)\}_{i,j=1,2}$, and prices $\{p_i^j\left(W(s), s\right), Q_i\left(W(s), s\right), q_i\left(W(s), s\right), e\left(W(s), s\right)\}_{i=1,2}$, such that 1) Given the prices and the law of motion for the aggregate net wealth fraction, consumption allocation, asset holdings, and value functions solve each consumer's problem, and 2) Markets clear.

Numerical algorithm I provide a *global solution* of portfolio choice, which implies that equilibrium is known for the time periods with large shocks far from steady state and under the occasionally binding collateral constraints. It is necessary to solve the model globally, especially to address a sudden and large drop of gross capital flows as a result of large negative shocks during the financial crisis and study the asset holdings in response to the stochastic borrowing limits. I use the time iteration algorithm by Kubler and Schmedders (2003), which has been applied to other international portfolio choice models such as Stepanchuk and Tsyrennikov (2015).[12] In section D of the Appendix, I describe the algorithm in detail.

3 Complete markets: risk sharing intuition

In this section, the risk sharing motive under the complete markets setting is analyzed before moving on to the quantitative analysis of the full model with incomplete markets. A simplified model without collateral constraints and two-state economy is introduced, where closed-form solutions of portfolio choice can be derived. This section highlights the mechanism of bond portfolio choice that can insure households against output shocks. Moreover, it is shown that under complete markets setting, bond positions do not change across states, implying that incomplete markets are essential for the generation of gross flows. In this section, the model is analogous to the previous setup, so only the differences are briefly described.

Physical environment There are two countries, country 1 and 2, where each country $i = 1, 2$ produces a single tradable good y_i. Output processes are exogenous and follows a stochastic process with two possible states of *High* (y_H) or *Low* (y_L): $y_i \in \{y_H, y_L\}$, $y_H > y_L$. There are two symmetric states in the world economy, defined as s_1 and s_2, which belong to the set of all states $S = \{s_1, s_2\}$. In state 1 (s_1), country 1's output is *High* ($y_1 = y_H$) whereas country 2's output is *Low* ($y_2 = y_L$). State 2 (s_2) is the symmetric case where the output is *Low* in country 1 and *High* in country 2: $s_1 = (y_1 = y_H, y_2 = y_L)$, $s_2 = (y_1 = y_L, y_2 = y_H)$. In every time period, there is an equal probability of 0.5 that each state realizes, i.i.d.

Households' problem and Market clearing Households' problem is analogous to the main model, except that there is no collateral constraint in the gross liability. Market clearing conditions remain the same.

[12] While the method itself was pioneered by Kubler and Schmedders (2003), it is not simply a straightforward extension of that algorithm to a setup with two-way flows. In particular, my approach requires fine-tuning the time iteration technique due to the high sensitivity of the portfolio choice policy functions to the returns on bonds, which are determined by the endogenous exchange rate. This problem is further complicated by the presence of occasionally binding collateral constraints that result in frequent kinks in the portfolio choice policy functions.

Social planner's problem Social planner maximizes the sum of two countries' flow utilities with equal weights, subject to feasibility constraints of each state.

$$U^*(s) = \max_{\{c_i^j\}, i,j=1,2} u_1(c^1(s)) + u_2(c^2(s)) \tag{15}$$

$$s.t. \ c_i^1(s) + c_i^2(s) = y_i(s), \ i = 1, 2 \tag{16}$$

Complete market solution I first solve for the social planner's allocations, and then find the bond portfolio that decentralizes the first-best allocations. First order necessary conditions of the social planner characterize the optimal tradable consumption across households at each state. These allocations critically depend on the risk aversion and elasticity of substitution between *home* and *foreign* tradable goods. Following propositions describe the conditions that determine bond portfolios $\{a_i^{j\prime}\}, \ i, j = 1, 2$. All proofs are in the Appendix.

Proposition 1. *If excess returns of bond 2 to bond 1 in state 1 (s_1) is not zero, then there is a unique bond portfolio $a^{1^*} = (a_1^{1*}, a_2^{1*})'$ that decentralizes the social planner's allocations. Specifically,*

$$a^{1^*} = \frac{nx_1^*}{rx_1^*} \begin{bmatrix} e^*(s_1) \\ -1 \end{bmatrix} \tag{17}$$

where nx_1^* and and rx_1^* are the first best net exports and excess returns of bond 2 to bond 1, respectively, in country 1 and state 1 (s_1).

$$nx_1^* = p_1^{1^*}(s_1)[y_H - c_1^{1^*}(s_1)] - p_2^{1^*}(s_1)c_2^{1^*}(s_1), \ rx_1^* = (1 - q_2^*(s_1))e(s_1) - (1 - q_1^*(s_1))$$

Corollary 1. *Given an elasticity of substitution σ and all other parameters ρ, define first best net exports of country 1 in state 1 (s_1) as*

$$nx(\sigma; \rho) \equiv p_1^{1^*}(\sigma; \rho)[y_H - c_1^{1^*}(\sigma; \rho)] - p_2^{1^*}(\sigma; \rho)c_2^{1^*}(\sigma; \rho)$$

where x^* denotes the social planner's solution in s_1 for any variable x. Then, $sign(a_1^{1^*}) = sign(nx(\sigma; \rho))$.

Corollary 2. *Let σ^* such that $nx(\sigma^*; \rho) = 0$. Then, for any $\sigma < \sigma^*$, $a_1^{1^*} < 0$.*

If two tradable goods are complements with sufficiently low σ, then the demand for foreign goods increases in the event of high domestic output. As Figure 4 describes for an example economy, below a certain level of σ country 1 becomes a net importer in the state of high domestic output (s_1) under the social planner's solution. In order to support such allocation

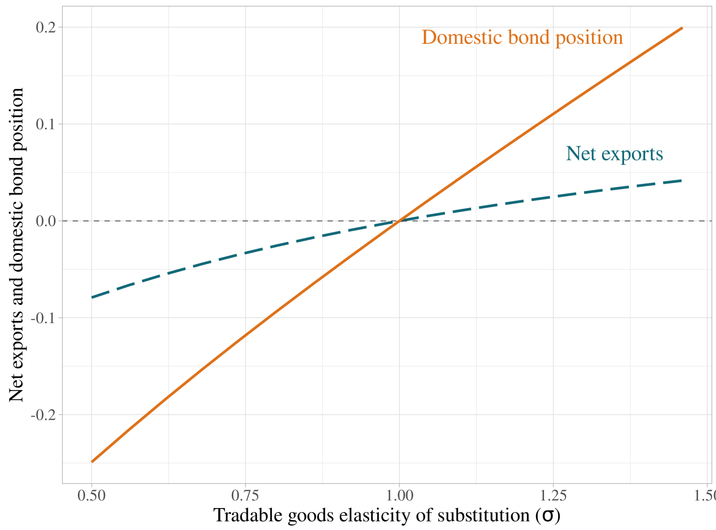

Figure 4: Net exports (nx_1^*) and domestic bond positions (a_1^{1*}) on σ

Note: All figures are for country 1, state 1 (s_1). Parameter values are specified in Appendix.

of consumption goods, country 1 households must take a short (negative) position in their domestic bonds (bond 1, $a_1^{1\prime}$) while taking a symmetric long (positive) position in the foreign bonds (bond 2, $a_2^{1\prime}$). In the event that foreign output is lower than the domestic one, returns on foreign bonds (bond 2) are higher than domestic ones (bond 1). In other words, excess returns (rx_1^*) on foreign bonds are positive in state 1. Therefore, by taking a long position in foreign bonds and simultaneously a short position in domestic bonds, households can finance their net imports in the event of high domestic output. Proposition 1 summarizes this relation between net exports, excess returns, and bond positions in a single equation. The size of bond positions are proportional to the size of net exports, and inversely related to the excess returns on foreign bonds relative to domestic bonds.

Finally, notice that there are no adjustments in the portfolio because there is a unique bond position that is common to all states. Therefore, there are no gross flows in the complete market case. This contrasts to the full model with incomplete financial markets, where households adjust their portfolios in each state and hence gross flows arise. While the direction and size of asset holding positions in the full model follow the intuition from the complete markets version, financial frictions prove to be essential in order to generate gross flows in a model. In the following Quantitative Analysis, the mechanisms of financial frictions and risk sharing in gross flows are studied in further detail using a calibrated model.

4 Quantitative Analysis

In this section, the full model introduced in section 2 is calibrated to the United States and the Rest of the World (RoW) in order to highlight two main mechanisms in the model: risk sharing and financial frictions. The main mechanisms are first inspected using policy functions of portfolio choice and impulse response functions. Next, gross flows and welfare implications driven by these two forces are analyzed quantitatively, using simulated results around the event of the Global Financial Crisis. Finally, baseline results are compared with alternative calibration settings in sensitivity analysis, focusing on key parameters such as elasticity of substitution between two tradable goods.

4.1 Calibration

Preference and output parameters The model is calibrated to the United States and Rest of the World, using data from 2000 to 2019. Two countries in the model are allowed to be asymmetric in parameter values, except for the following core preference parameters: discount factor (β), risk aversion (γ), and finally elasticity of substitution between two tradable goods (σ, also known as Armington elasticity). These parameter values are taken from the literature, as summarized in Table 1, following widely used values in international macro literature for the discount factor and risk aversion.[13] It should be noted that the direction and magnitude of asset holdings critically depend on above parameters, as shown in the previous Complete markets section. For that reason, in the Sensitivity Analysis section following baseline results, further discussions regarding quantitative and qualitative relationship between these parameters and main results are provided.

Other parameters are mostly estimated from the US and RoW output data. The home bias parameters in the US ($\omega_H^1 = 1 - \omega_F^1$) and RoW ($\omega_H^2$) are calibrated using the average of imports and exports to GDP in the US against RoW during the sample period, which are 16% and 13%, respectively. Parameters for the output series, including persistence and variance-covariance matrix of exogenous shocks, are estimated based on the US and RoW GDP series.[14] Here, the output parameters are allowed to be asymmetric across countries.

[13]In the literature, estimations of Armington elasticity, for example, range from as low as 0.1 to around 2 for the G-7 countries (Hooper et al., 2000), which is in line with the usual range for the macro literature. For example, Corsetti et al. (2008) follow the estimated value of 0.85 by Mendoza (1991), which is based on developed economies, and Stockman and Tesar (1995) sets it equal to 1.

[14]All series are in log and linear trends are used. More details of the data construction can be found in Appendix H.

Table 1: Parameters

Parameter	Description	Value	Target/Source
Preference			
β	Discount factor	0.980	Steady state interest rate 2%
γ	Risk aversion	1.000	Literature
σ	Elasticity of substitution	0.900	Heathcote and Perri (2002)
ω_H^1	Home bias, US	0.840	Imports to GDP, US
ω_H^2	Home bias, RoW	0.870	Exports to GDP, US
Output			
ρ_1	Persistence, US	0.928	Estimated
ρ_2	Persistence, RoW	0.886	Estimated
σ_1	Shock sd, US	0.019	Estimated
σ_2	Shock sd, RoW	0.025	Estimated
ρ_{12}	Cross-country shock correlation	0.395	Estimated
Collateral constraint			
$\bar{\chi}_{ss}$	Steady state level	0.020	US Flow of Funds
$\bar{\chi}_1$	Initial level	0.017	US Flow of Funds
κ_{ss}	Steady state asymmetry	1.268	US Flow of Funds
κ_1	Initial asymmetry	1.201	US Flow of Funds
ζ	Crisis shock	0.945	US Flow of Funds

It is noteworthy that the home bias in preferences and asymmetric output parameters partially address some of the well-known international macroeconomic puzzles in the context of the US external positions. For example, home bias in final goods composition is a part of the key characteristics in explaining the asset home bias puzzle (Heathcote and Perri, 2013).[15] Also, it is known that the US net income flows are positive even though it holds a substantially negative net investment position especially from 2000 to 2007 (the income puzzle). Another related puzzle is that there is a large gap between the cumulated current accounts and reported net international positions in the US (the position puzzle, Gourinchas and Rey, 2007b). Asymmetries in output and home bias parameters across two countries in equilibrium lead to differential returns in foreign and domestic bonds, and therefore partially address the income and position puzzles. In other words, holding one unit of RoW bonds gives a higher return than US bonds on average, due to the endogenous real exchange rates in equilibrium.[16]

[15] Moreover, the model embeds the home bias in assets. In particular, it is assumed in the model that the entire equity is held by domestic households, which abstracts from the equity choice while being consistent with the home bias puzzle in assets.

[16] More specifically, there is a higher demand in RoW goods than in US goods, due to the asymmetry in home bias parameters. This results in the real exchange rate e, the price of RoW goods to the US goods,

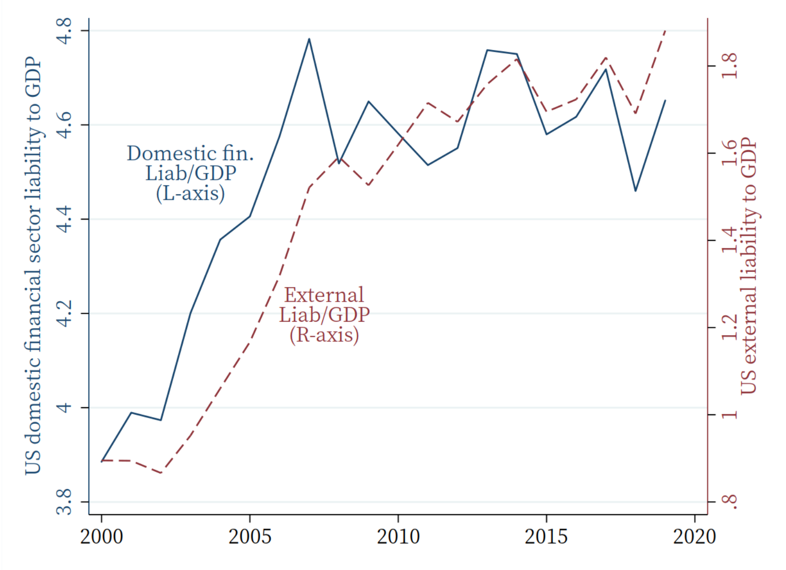

Figure 5: US domestic financial sector and external liability to GDP ratio

Note: Domestic financial liability is calculated from the US Flow of Funds, domestic financial sectors, total liabilities. External liabilities are sourced from Bureau of Economic Analysis, International Investment Positions, Liabilities.

Collateral constraints The functional form of collateral constraints is composed of three parts: a global deterministic trend (trend), country-specific deterministic trends (asymmetry), and global stochastic crisis shocks (crisis). Concretely, collateral constraints in country j and time t (on a state node s^t), denoted as $\chi^j(s^t)$, can be formulated as:

$$\chi^j(s^t) = \bar{\chi}_t \times \kappa_t^j \times \zeta(s_t; s_{t-1}) \tag{18}$$

where $\bar{\chi}_t$ is the trend in time t, κ_t^j is the asymmetry in country j time t[17], and $\zeta(s_t; s_{t-1})$ is the *crisis* shock that follows a Markov chain process with two values: Normal ($\zeta = 1$) and Crisis ($\zeta < 1$).

In order to calibrate the above function, the US domestic financial sector's liability data from the Flow of Funds was used for the deterministic trend and crisis shock parts of collateral constraints.[18] The reason for using the domestic financial sector's balance sheet data is because

to be larger than 1 in equilibrium. Bond returns depend on the real exchange rates, also known as valuation effects.

[17] κ_t^2 is set to 1 as a normalization for all time t, so that κ_t^1 is asymmetry in country 1 compared to country 2

[18] Flow of Funds table, Domestic financial sectors; total liabilities (FL794190005.A)

a majority of gross capital flows are driven by *bank flows*, as discussed in Data section (Section 1.2). Also, as depicted in Figure 5, patterns of external liability to GDP match closely to those of domestic financial sector's liability to GDP. In order to design functional forms that reflect the patterns of domestic financial liability to GDP, the collateral constraints in the model are composed of both deterministic trend and stochastic crisis shock. Finally, the asymmetry parameters are calibrated using the International Investment Position of the US, in particular the relative size of US external liabilities to assets at the beginning and the peak periods as in the trends. Such choice of calibration is motivated by the Global Imbalances literature, reflecting differences in financial development between the US and the RoW.[19]

More specifically, over the sample period from 2000 to 2019, it is assumed that the trend and asymmetry parts of constraints $\bar{\chi}_t$ evolve at a constant growth rate starting from 2002[20] up until 2007, and then stays at the "steady state" level thereafter. The ratio of initial to steady state trend level ($\bar{\chi}_1/\bar{\chi}_{ss}$) is calibrated by taking a ratio of the domestic financial sector liability to GDP in 2002 and 2007, which is 83.1%.[21] The initial and steady state asymmetry in country 1, κ_1^1 and κ_{ss}^1, are based on the ratio of US external liabilities to assets in 2002 and 2007, respectively. The last component of collateral constraints, the magnitude of crisis shock ζ, is calibrated based on the ratio of crisis onset (2008) to peak time (2007) financial sector liability to GDP, which is 94.47%. The transition probability of Crisis shock is based on the duration and frequency of financial crisis over the sample period: once in 20 years, lasting for 2 years.[22]

4.2 Equilibrium portfolios and gross flows mechanism

Using a calibrated model, the main mechanism of consumption risk sharing and financial frictions is analyzed in this section. First, bond policy functions with and without collateral constraint shocks are compared. This comparison highlights the role of financial frictions on the asset and liability positions of households. Then, impulse response functions under a one standard deviation shock on the country 1's output are analyzed. This analysis emphasizes how equilibrium bond holdings smooth the households' consumption over time, and shows the direction and magnitude of gross flows on the impact of negative output shocks. In both bond policy functions and impulse response functions, parameters are set as explained in the

[19] For example, Mendoza et al., 2009 and Caballero et al., 2008a show that differences in financial development can explain the observed rise in US current account deficits.

[20] The starting year is selected so that the growth of collateral constraints starts after the 2001 recession.

[21] The steady state level of trend is calibrated so that the model portfolio is unconstrained at the "zero-shock" state. The concept of zero-shock state is further elaborated in the following Section 4.2.

[22] Transition matrix is such that $Prob(\zeta < 1|\zeta = 1) = 0.1$, and $Prob(\zeta = 1|\zeta < 1) = 0.5$

previous section, except that the cross-country shock correlations in output are set to zero. By setting a zero cross-correlation, impulse response functions highlight the effects of output shocks more clearly than the benchmark calibration with a positive cross-correlation.

Bond policy functions In the model, bond policy functions hold the key channel that links output risk sharing, collateral constraints, and gross capital flows. One of the natural states to inspect bond policy functions is the *zero-y-shock state*, where there are zero shocks for all the exogenous output series (y_i), conditional on a given level of collateral constraint (χ)[23]. In equilibrium, consumers save in foreign bonds and borrow in domestic bonds for any given collateral constraint. Also, under zero-y-shock state and without any collateral constraint shock, there is a certain net wealth fraction w_0 such that consumers save and borrow the same amount of bonds simultaneously, holding zero net positions.[24] This implies that if the two countries start at the zero-net-position wealth fraction w_0 and the zero-y-shock state, then they remain at the same state until an exogenous shock hits the output or collateral constraints. Henceforth, the zero-net-position wealth fraction w_0 together with zero-y-shock state without collateral constraint shocks are set as a natural starting point for the following impulse response functions as well as simulations in the next subsection.

In Figure 6, policy functions of household 1 for both domestic (dashed lines) and foreign (solid lines) bonds are illustrated, conditional on without (left) and with (right) collateral constraint shocks. Notice that *negative* values of domestic assets ($-a_1^{1\prime}$) are plotted in Figure 6, so that they are directly mapped to the amount of liabilities. In other words, households borrow in domestic bonds for any level of net wealth or collateral constraint, and this equilibrium portfolio allows a direct mapping of external liabilities to the absolute amount of domestic bond holdings.

Absent adverse collateral shocks, households increase the amount of savings in foreign bonds (assets) and reduce borrowings in domestic bonds (liabilities) as the net wealth fraction w increases. As a result, they increase their net savings with higher relative wealth. On the other hand, when the collateral constraint tightens due to a negative shock, the amount of borrowings by relatively poorer households ($w < w_0$) are limited to a smaller fraction of their equity holdings. This results in a kink of liabilities and assets at the zero-net-position

[23]This is a similar concept to the Risky Steady State proposed by Coeurdacier et al. (2011) under the symmetric setting and conditional on a certain level of collateral constraint. Since collateral constraints do not have a steady state in this model, an alternative term zero-y-shock state is coined in this paper.

[24]The level of w_0 depends on the output series and utility parameters of both countries. In particular, if two countries are perfectly symmetric, it will be at the net wealth fraction of $w_0 = 0.5$.

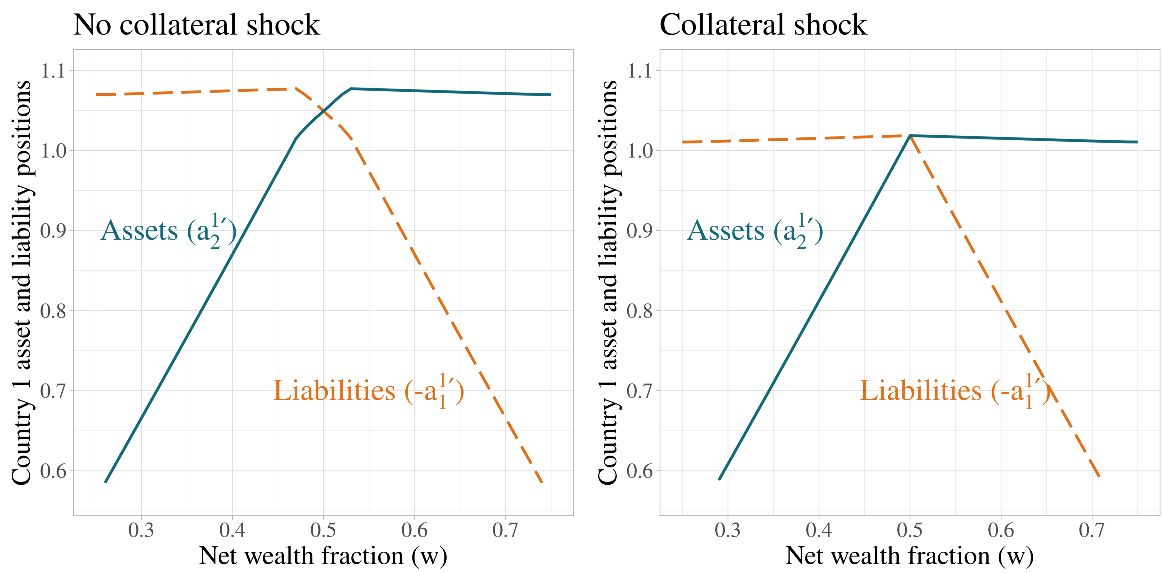

Figure 6: Asset and liability position policy functions of country 1 at zero-y-shock state, without (left) and with (right) collateral constraint shocks.

wealth level w_0. When households are relatively poor, they borrow to the maximum amount under the collateral constraint, while as they turn relatively rich, they reduce the amount of liabilities compared to the assets in foreign bonds. The net savings increase with the wealth subject to a negative collateral constraint shock as well, but the gross positions are bounded by either home or foreign country's collateral constraints.

Impulse response functions Dynamic responses of the consumption, bond portfolios, and prices can be best inspected through impulse response functions. Figure 7 describes responses of endogenous variables to a negative shock to the country 1 output (y_1) in time 1, starting from a zero-y-shock, without collateral constraint shock, and at zero-net-position wealth level w_0 in time 0. All other exogenous shocks are set to be zero.

As Figure 7 shows, with a decrease in domestic output, domestic goods become more valuable than foreign goods in country 1. This leads to a cheaper basket of foreign goods compared to the domestic basket and hence a decline of real exchange rates (panel (e)). These movements in the real exchange rates result in a lower net wealth fraction for country 1 consumers (panel (d)), because the value of savings in foreign bonds decline while the value of borrowings in domestic bonds increase. Both households reduce their consumption of good 1 by a similar magnitude (panel (b)), as a response to the negative shock in output. Households in country 1 also cut down on their bond holdings in absolute values for both domestic and foreign bonds (panel (c)), which reduces their exposures to the changes in net wealth due to exchange rates.

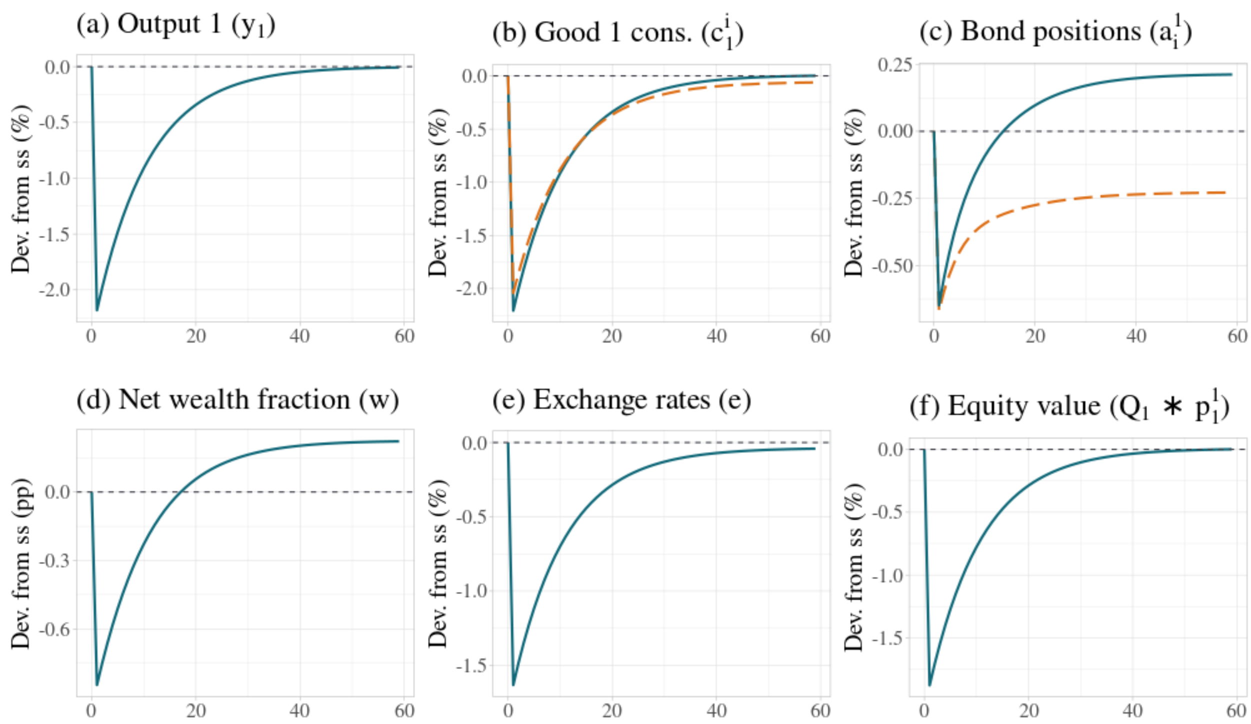

Figure 7: Impulse response function, 1 sd. shock on country 1 output

If the households do not scale down the amount of assets in foreign bonds and liabilities in domestic bonds, their net wealth would further deteriorate because of a decline in exchange rates. In order to prevent excessive declines in the net wealth, households reduce the size of their external balance sheet in response to a decrease in exchange rates. Afterwards, asset positions recover faster than liabilities, but eventually both bond positions return to their original levels.

Finally, equity value (panel (f)) decreases simultaneously with the lower output, which implies that the borrowing limit as a fraction of equity value tightens on the impact of a negative output shock. Despite an appreciation of domestic tradable goods price (p_1^1), the equity price (Q_1) drop dominates in its magnitude, and the value of collateral decreases as a result. This response in the equity value survives under a negative collateral shock as well, which exacerbates the tightening of collateral constraints even further when collateral constraints and output are hit by a negative shock simultaneously.

As the impulse response functions show, households decrease their holdings of both domestic and foreign bonds on the impact of negative output shock. These adjustments result in *negative* gross capital flows, which account for the changes in both asset and liability positions.

The key mechanism is the changes in net wealth through the endogenous movements in exchange rates and the bond positions. When foreign and domestic tradable goods are complements, as well as non-tradable and tradable goods, it is optimal for the households to reduce their consumption on both foreign and domestic tradable goods when the domestic output is hit by a negative shock. Therefore, knowing that exchange rates would decline when such negative shocks hit the domestic output, households in equilibrium save in foreign bonds and borrow in domestic bonds. Moreover, they adjust the amount of bond holdings dynamically in order to optimally control the changes in their net wealth in response to the movements of exchange rates and output shocks.

4.3 US Great Recession

In order to study the quantitative importance of risk sharing and financial frictions on gross flows and analyze welfare implications, the model is solved for the years of 2000-2019 which includes the Great Recession in the United States. Real GDP series of the United States and the Rest of the World output series are taken as exogenous states (y_i), and used as an input of the model simulation. The model is initiated at the zero-y-shock state and zero-net-position wealth fraction level, as described in the previous section. In the sample period, collateral constraints remain at the trend level except for the years 2008-2009, when they are hit by crisis-level negative shocks.

The main exercise is first to generate the equilibrium portfolios on domestic and foreign bonds as an endogenous response to the exogenous output shocks and hence compare gross flows in the model and the data. Then, quantitative importance of two channels, risk sharing and collateral constraint, is analyzed by comparing the benchmark simulations to the counter-factual ones where each channel is shut down. Finally, welfare implications especially during the financial crisis are highlighted as part of the main results. These welfare implications are revisited in the following Sensitivity Analysis section (Section 4.4) with different levels of Armington elasticity.

Output process In order to construct the series of exogenous output in the US and Rest of the World, the US real GDP and real exchange rates of the US dollars are used. For the US, annual real GDP series from Bureau of Economic Analysis is de-trended linearly. In order to construct the RoW output series, Real Effective Exchange Rates (REER) obtained from FRED[25] is used together with the detrended US output series. More specifically, a series of

[25] Bank for International Settlements, Real Broad Effective Exchange Rate for United States [RBUSBIS], retrieved from FRED, Federal Reserve Bank of St. Louis.

(detrended) log RoW output is constructed as a sum of detrended US output and a fraction of log REER.[26] Figure 17 summarizes results of log output series following these procedures.

The main reason for such construction of the RoW output series is to leverage the model mechanism that endogenously generates the real exchange rates so that simulated real exchange rates match well with the data. More specifically, as the impulse response functions (Figure 7) show, model exchange rates appreciate in response to a negative shock in domestic output. That is, with the above construction of the RoW output series, it is assured that real exchange rate fluctuations mimic the patterns in the data well. Therefore, this RoW series can be viewed as an "induced" RoW series through the lens of real exchange rates in the US. Since the portfolio choice critically depends on the endogenous movements of exchange rates, it is important that the model-generated exchange rates can match those observed in the data.

Gross capital flows Using the exogenous output series as input of the model, the main results are an endogenously generated series of gross capital flows. In the model, asset and liability flows are calculated as today's bond purchases net of the previous period's. Since households under the benchmark calibration save in foreign bonds and borrow in domestic bonds, asset and liability flows are derived from foreign and domestic bond positions, respectively.[27] Gross flows are equal to the sum of asset and liability flows, and are endogenously generated from the model as the households optimally adjust their bond holdings according to their net wealth fractions, output shocks, and collateral constraints. The data series for the comparison is sourced from the Bureau of Economic Analysis, International Transactions data, an analogous series as in Figure 1 of the Data section but at annual frequency.[28]

[26]Formally,

$$\log y_{2,t} = \log y_{1,t} + \omega_{REER} \log REER_t$$

The REER series normalized so that the average is equal to 1 over the sample period. An increase of REER implies appreciation of the US dollar. In the baseline calibration, the fraction ω_{REER} is set to 34.5%, so that the ratio of σ_1/σ_2 matches with Table 1.

[27]Asset and liability flows for country 1 are:

$$\begin{aligned} asset\_flows^1(s^t) &= e(s^t)[q_2(s^t)a_2^1(s^t) - q_2(s^{t-1})a_2^1(s^{t-1})] \\ liab\_flows^1(s^t) &= -\left[q_1(s^t)a_1^1(s^t) - q_1(s^{t-1})a_1^1(s^{t-1})\right] \end{aligned}$$

[28]U.S. International Transactions, the sum of Net U.S. acquisition of financial assets excluding financial derivatives (net increase in assets / financial outflow (+)) and Net U.S. incurrence of liabilities excluding financial derivatives (net increase in liabilities / financial inflow (+)) over GDP. Note that the total gross flows, including both equity and debt flows, are used in the comparison.

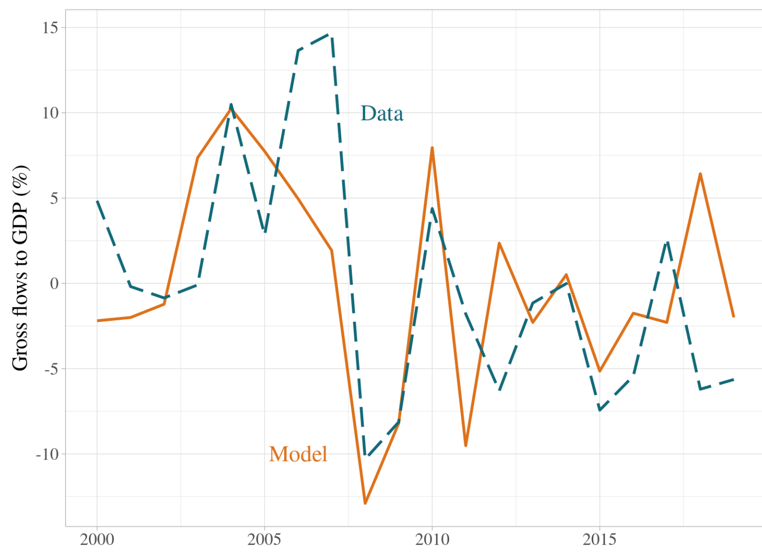

Figure 8: Gross capital flows, data vs. model

Note: Both model simulated and data gross capital flows are de-meaned over the sample period. Data source is Bureau of Economic Analysis and author's calculations.

The model simulated gross flows closely follow the patterns of data observations, as shown in Figure 8 with the comparison of the model (solid) and data (dashed) series, both demeaned.[29] This suggests that the two main channels of the model, risk sharing and financial frictions, can explain a large part of observed gross flows. In particular, a loosening of collateral constraints until 2007 matches with an increase of gross flows. A tightening of collateral constraints due to the *crisis* shocks combined with a sharp drop in output corresponds to a significant drop in gross flows. It is noteworthy that the model is also capable of matching a rebound of gross flows once the collateral constraints return to the trend level in 2010, followed by a relatively slow recovery.

In addition, magnitudes and key characteristics of simulated gross flows match closely to the data. Standard deviation of gross flows to GDP in the model is 6.2%, which explains nearly 90 percent of the data standard deviation (7%). The model also generates the boom-

[29] In the quantitative exercise, demeaned series are compared for two reasons. First, the main model focuses on the dynamics of allocations and prices *without* a balanced growth path in the economy. If the external portfolio positions grow along with the growth in the real economy, there will be positive capital flows without risk sharing nor financial frictions. This is beyond the scope of this paper. Secondly, there has been a secular increase in the US liability positions, which led to higher liability flows than asset flows persistently over the sample period. There are many papers that discuss global imbalances both empirically and theoretically (Mendoza et al. 2009, Gourinchas and Rey 2007a, Lane and Milesi-Ferretti 2007, Caballero et al. 2008b, Maggiori 2017). This paper leaves the combination of endogenous asymmetry (imbalances) and fluctuations of gross flows to the future research.

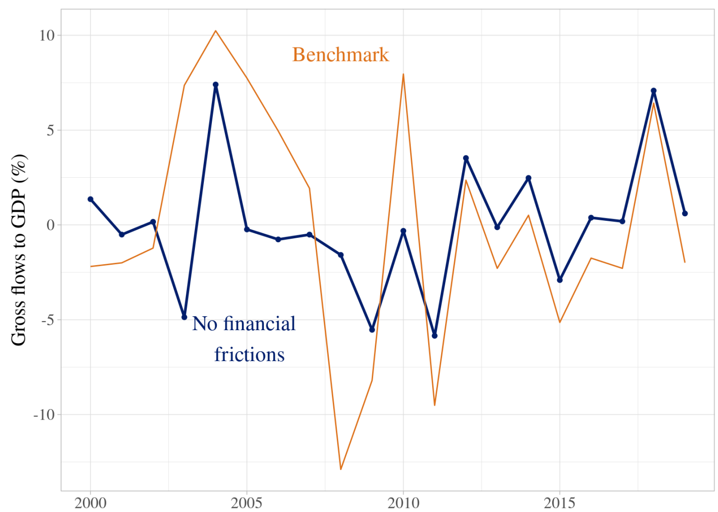

Figure 9: Gross capital flows, benchmark vs. no financial frictions

Note: Both model simulated and data gross capital flows are de-meaned over the sample period.

bust patterns observed in the data, including a similar magnitude of peak-to-trough collapse in gross flows during the Great Recession. Moreover, asset and liability flows are highly correlated to each other in both model (99%) and data (94%), based on Pearson's correlation coefficient. Analogous figures for the asset and liability flows can be found in the Appendix I, Figure 18. In the above benchmark model, two channels are at play in generating the simulated gross flows: risk sharing and financial frictions. In the following, quantitative importance of each channel is analyzed by shutting down parts of the channels and comparing the simulated results.

Decomposition: Risk sharing vs. Financial frictions In order to decompose the movements of gross flows into the risk sharing and financial friction channels, in a counterfactual exercise all financial frictions are shut down and an analogous simulation is executed as in the benchmark. More specifically, it is assumed that there is no trend growth of collateral constraints but instead they remain at the "steady state" level throughout the sample period. In addition, there is no crisis shock realization during the years of 2008-2009, while the agents in the model acknowledge the possibility of such realization. Figure 9 compares the benchmark gross flows (orange solid line) to the "no financial frictions" counterfactual scenario (navy solid line with dots).

Two main observations can be drawn from the counterfactual exercise of shutting down financial frictions. First, more than half of the collapse of gross flows during the financial crisis can be attributed to the financial frictions channel, as opposed to the risk sharing channel. Especially, in 2008, output shock was negative but modest compared to the following year, but the gross flows experienced a dramatic collapse. This implies that the risk sharing channel alone cannot explain the dynamics of gross flows during the financial crisis, and the counterfactual exercise demonstrates it.

Another main observation from the counterfactual exercise is that a steady relaxation of borrowing constraints can account for most of the boom in gross flows from 2002 to 2007. While the US and the rest of the world enjoyed positive output shocks in the early 2000's, the magnitude of output shocks does not match with the rise of gross flows observed in the data. Instead, the model suggests that an easing of international financial frictions can explain most of the increase in both asset and liability flows, especially up to 2006.

Finally, a similar counterfactual exercise where there is no realization of crisis shock during the Great Recession can be found in the Appendix. This "no crisis shock" exercise sheds light on the impact of stochastic financial frictions, while the trend growth of collateral constraint remains as in the benchmark case. Figure 19 in Appendix shows analogous results for the years 2008-2009 as in the "no financial frictions" exercise. In the absence of crisis shock realizations, gross flows to GDP would decline by less than half of the magnitude in the baseline simulation.

Welfare analysis Equipped with the model that generates gross flows that match well with the data observations, a natural next step is to use the model simulated results to study the welfare implications of international positions and gross flows. In this section, the welfare effects are analyzed by comparing the final consumptions of the benchmark scenario with the "single-bond" environment. More specifically, in the single-bond economy, households are allowed to trade only the Home (calibrated to the US) bonds, in an otherwise similar environment as in the benchmark. Notice that only net positions will be generated in the single-bond economy, and hence there will not be any gross flows. Hence, the comparison of benchmark economy consumption with the single-bond economy highlights the welfare effects of gross flows compared to the more traditional net flows. In addition, consumptions in the two extreme versions of asset markets, namely complete markets and financial autarky where only goods trading is allowed, are compared. Since both the benchmark model and the single-bond economy fall in between the complete market and financial autarky environment, the maximum range of changes in consumption can be derived from presenting the two extreme versions of the model. Full descriptions of the single-bond, complete markets and financial autarky economies can be found in the Appendix.

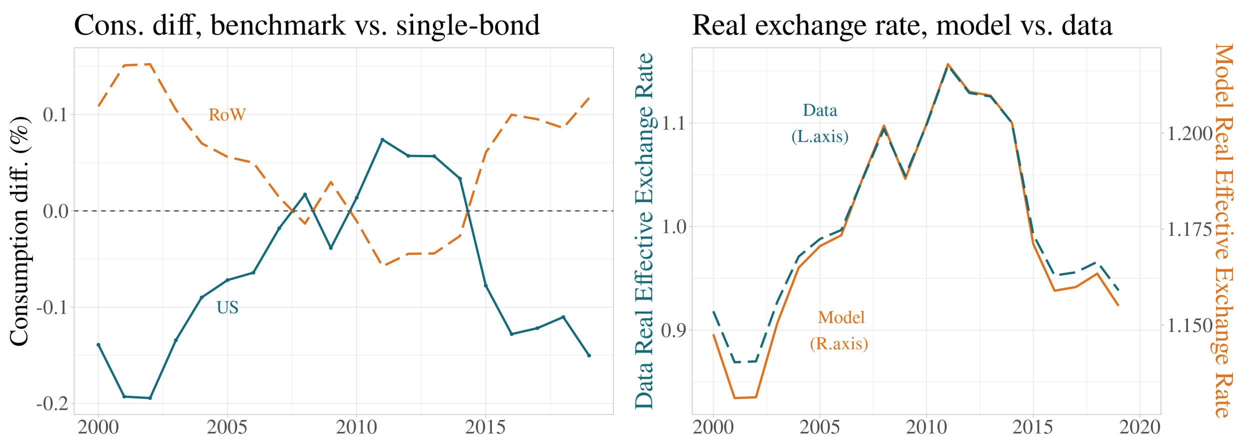

Figure 10: Consumption differences in the benchmark vs. single-bond model (left) and real exchange rates in the data vs. model. (right)

Note: Left figure y-axis is differences in log final consumption of the benchmark model and the single-bond model, in the US (solid) and RoW (dashed). Right figure is real exchange rates in data (dashed, left axis) and model (solid, right axis).

The key difference between the consumption series between the single-bond and benchmark model is driven by the exchange rates. The left panel of Figure 10 depicts the aggregate consumption differences of the US (solid green) and the Rest of the World (orange dashed) in the benchmark model compared to the single-bond economy. A positive value implies that the level of consumption in the benchmark model is higher than the level in the single-bond model. On the right panel of the same figure, I compare the model real exchange rates (orange solid line) generated from the model to the data counterpart (green dashed line). Comparing these two panels, I show that fluctuations in the real exchange rate mostly account for the differences in consumption between the single-bond and the benchmark economy.

More specifically, when the Home currency (calibrated to the US dollars) appreciates, the US households experience lower consumption because they transfer wealth to the Rest of the World. Compared to the single-bond economy, for example, in the benchmark model the US households consumed less in 2009 at the height of the crisis due to the appreciation of the US dollar. It is precisely due to the cross-border portfolio positions in the model, where the US borrows in its own currency and invests in the foreign currency. When the USD appreciates, the value of liabilities in USD goes up relative to the value of assets in foreign currency. This results in a decrease in net wealth in the US. In the two-country economy, a decrease of net wealth in the US is equivalent to an increase of net wealth in the RoW, which is a wealth transfer from the US to the RoW. As a result of this wealth transfer from the US to the RoW during the bad times, the RoW households can enjoy higher consumption (the concept also known as "exorbitant duty", Gourinchas and Rey 2022). The same mechanism works in the opposite direction when the US dollar depreciates - US households enjoy higher consumption due to the wealth transfer from the RoW households.

A crucial role of gross flows is to control the magnitude of wealth transfers in response to the exchange rate fluctuations. For example, if households double the asset and liability positions, then the amount of wealth transfer across countries doubles to a 1% fluctuation in exchange rates. Therefore, by adjusting their international investment positions dynamically, and therefore generating gross flows, households optimize the degree of cross-country risk sharing every period. It should be emphasized that the wealth transfer from the US to the RoW in 2009 is a result of optimal risk sharing; since two tradable goods are gross complements, households would like to reduce the consumption of foreign goods whenever the domestic output decreases.Financial frictions due to a tightening of collateral constraints restrict the degree of international risk sharing whenever the size of liabilities cannot reach the desired level.

Further comparisons with two other extremes of the financial market completeness, namely complete markets and financial autarky, demonstrate the relationship between the financial frictions and the degree of international risk sharing. In Figure 20 of the Appendix, Financial autarky (left panel) shows a similar direction of consumption differences as those compared to the single-bond, but more magnified. Finally, comparison with the complete markets in the same figure (right panel) shows that the US households loses about 6% of consumption compared to the benchmark model due to the financial frictions, an order of magnitude larger than the financial autarky environment. This shows that the benchmark economy consumptions are closer to the levels in financial autarky than in the complete markets, implying a significant degree of financial frictions in the economy.

Real exchange rates, trade and net flows As explained in the welfare analysis, real exchange rates play a key role in the international risk sharing especially when combined with assets and liabilities denominated in different currencies. Crucially, welfare gains or losses compared to the single-bond market depends on the depreciation or appreciation of the Home currency, respectively. Therefore, it is important that the exchange rates that are

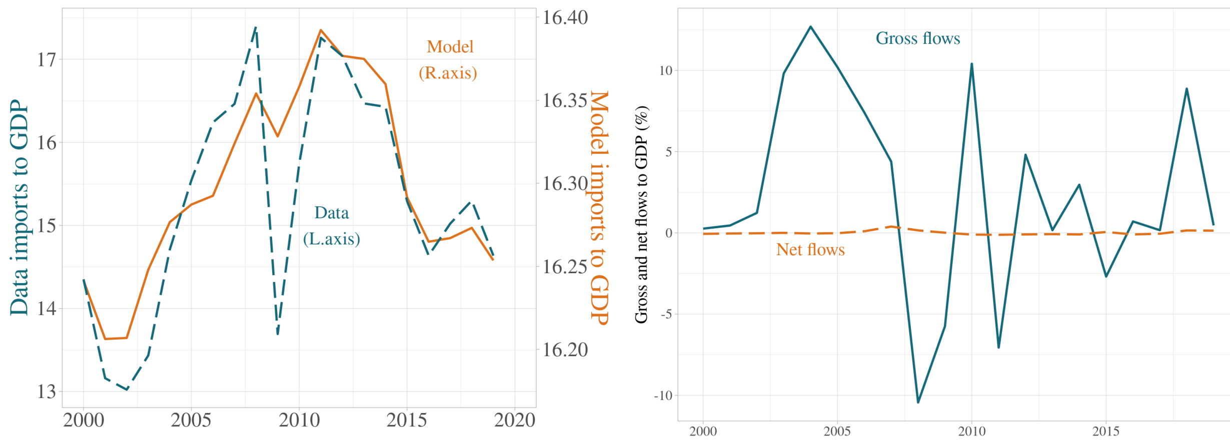

Figure 11: Imports to GDP and net flows in the model and data

endogenously generated in the model match well with the data observations, especially the dynamics of exchange rate fluctuations. The right panel of Figure 10 shows that the model generated exchange rates mimic the movements of the data series very well, even though the magnitude of fluctuations fall short of the data observations.[30] Because the key determinant of exchange rates in this model is the relative output of Home and Foreign, where Home currency appreciates when the Home output is lower than the Foreign output, the simulated dynamics of real exchange rates match nicely with the data series by the design of RoW output series (see Section 4.1).

While the main focus of this paper is on the gross flows, trade and net capital flows generated in the model also feature some of the key characteristics that are observed in the data. In Figure 11, the model simulated series of imports to GDP (left panel) and net flows (right panel) are compared with the corresponding data series. The imports to GDP time series in the model follows closely to the data dynamics, albeit smaller in the magnitude of fluctuations[31] and the collapse of the volume during the Great Recession is not as dramatic as in the data. In the Appendix Figure 21, a high correlation of imports and exports in the model can be observed, which is consistent with data as well.[32] Finally, comparing the patterns of

[30]As the impulse response functions show in Figure 7, fluctuations in exchange rates in the model are *less* than output fluctuations. It is because exchange rates are in essence a ratio of relative prices across two countries, which are in equilibrium marginal utilities. In the data, volatility of exchange rates is much bigger than output volatility. In order for the model to match the observed level of volatility in exchange rates, it needs to deviate from a classic way of deriving exchange rates.

[31]Smaller magnitude can be partly attributed to smaller fluctuations in exchange rates. As the imports to GDP highly correlated with the exchange rates, more fluctuations in exchange rates would lead to higher variations in imports to GDP.

[32]Note that while imports and exports are highly correlated with each other, it is possible that imports to GDP and exports to GDP series behave differently, as the Appendix figure shows. More specifically,

net flows to gross flows generated from simulated results, the model also features much more volatile gross flows compared to net flows, which is consistent with the data observations in Section 1. Figure 11 on the right panel plots both gross and net flows from the benchmark model, where net flows are close to zero throughout the sample period and the magnitude of fluctuations is much smaller compared gross flows.[33]

4.4 Sensitivity Analysis

In this section, the benchmark model results of gross flows and welfare analysis are compared to both higher and lower levels of Armington elasticity, and a higher level of risk aversion.

As the previous section of complete markets (Section 3) and calibration (Section 4.1) mentioned, Armington elasticity of two tradable goods plays a critical role in determining external portfolios and hence movements of gross flows and international risk sharing. With a lower level of Armington elasticity (0.7)[34], the levels of bond holdings are higher, and the corresponding movements of gross flows are larger in magnitudes compared to the benchmark. The welfare gains and losses for the US during the sample period are amplified under the lower level of Armington elasticity (left panel of Figure 12, orange dashed line), which are calculated in an analogous fashion to the previous section by subtracting the single-bond model aggregate consumption from the benchmark model with a lower elasticity. This amplification is due to the fact that households now hold larger amounts of both external assets and liabilities, and a 1 percent fluctuations in exchange rates bring larger transfers in wealth compared to the benchmark elasticity.

On the other hand, with a higher level of Armington elasticity (1.1), the external asset and liability holdings switch their directions compared to the benchmark elasticity. Specifically, since foreign and domestic tradable goods are substitutes under the Armington elasticity higher than 1, households save (long position) in domestic bonds and borrow (short position) in foreign bonds. This is because when a negative shock hits the domestic output, for example, households now want to substitute their home good consumption with the foreign one by importing more. In order to do so, they save in domestic bonds whose value goes

exports to GDP moves in the inverse direction with the real exchange rates.

[33]Compared with the data series of net flows, the model net flow fluctuations are orders of magnitude smaller. This is because the model does not target the initial net flows in the US, which would require addressing the Global Imbalances literature quantitatively. A joint quantitative study of net flows and gross flows is left for future research.

[34]While all other parameters remain the same, collateral constraints change with the elasticity because the global minimization as explained in the Calibration section (4.1) applies to each set of parameters. Full description of the collateral constraint parameters can be found in Appendix.

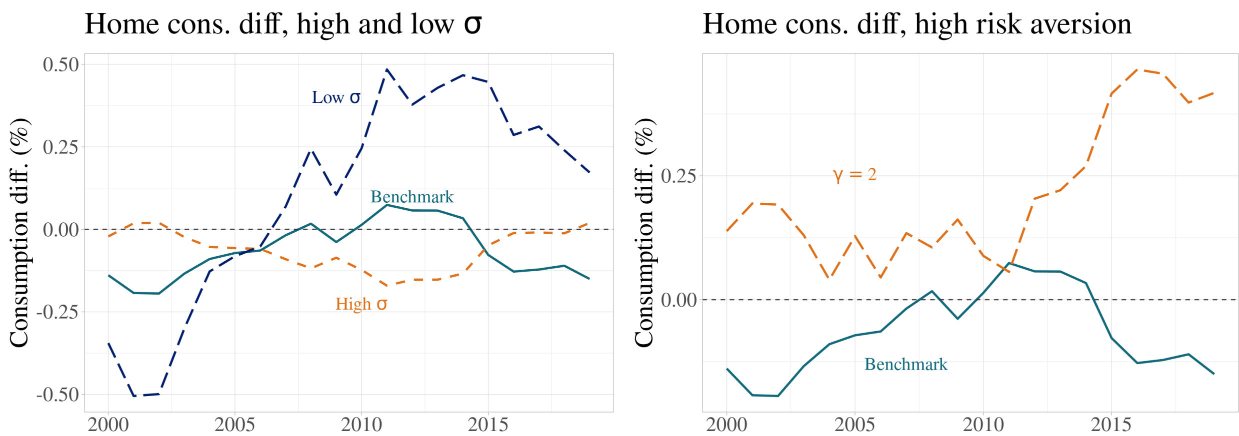

Figure 12: Benchmark - financial autarky aggregate consumption in the US, Armington elasticity of 1.1 (left) and 0.7 (right), compared to 0.9

up during bad times due to a real appreciation. Due to this switch in external bond positions, the wealth transfer and hence welfare gains (losses) during the sample period mirror the benchmark results. As the left panel of Figure 12 shows, RoW now transfers wealth to the US under a higher elasticity during the Great Recession. While the welfare gains and losses change their direction under a higher elasticity, the magnitude of gross flows and welfare gains/losses remain relatively similar to the benchmark model. However, with an even higher elasticity than this example case of 1.1, the magnitudes of both gross flows and welfare gains (losses) increase, similarly to the lower elasticity case of 0.7. It is noteworthy that the portfolio positions and exchange rates in the high Armington elasticity case resemble the developing economies, as discussed in the earlier section (Section 1.3).

Finally, when the risk aversion parameter increases to $\gamma = 2$, there is a similar welfare effect as with high σ can be observed. With high enough Armington elasticity (0.9 in the benchmark), an increase of risk aversion to 2 turns Home and Foreign tradable goods into gross substitutes. This implies that welfare effects with respect to real exchange rates move in the opposite direction compared to the benchmark simulation, as with the high σ case. In other words, now US households save in US dollars and borrow in foreign currencies under higher risk aversion. An appreciation of the US dollars at the end of the sample period, for example, increases the consumption of US households compared to the single-bond economy due to the international investment positions.

5 Conclusion

This paper shows that international capital flows transfer significant amounts of wealth across countries, and hence play a critical role in cross-border risk sharing. In this paper, the main hypothesis is that households share tradable output risks across the border by holding international assets and liabilities, subject to financial frictions. These risk sharing motives and financial frictions are the two main channels that generate the movement of international portfolio positions, and hence gross capital flows. By building a quantitative model, this paper shows that the gross flows benefited the US during the Great Recession, while the severe financial frictions in 2009 hampered such risk sharing. This paper successfully generates endogenous gross flows from the model, which enables quantitative analysis of capital flows on welfare and decomposes the importance of risk sharing and financial friction channels. Admittedly, this paper builds a stylized model of two countries and two bonds, in order to tractably solve the system using a global solution method. Potential expansions of this paper, such as the inclusion of equity portfolio and asymmetric financial frictions, come at the cost of reduced tractability and more computation power, and are left to future research.

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Appendices

A Risk sharing and gross flows

Data All data sources are from IMF. From International Financial Statistics (IFS), nominal and real GDP in domestic currency, real final consumption, and exchange rates of domestic currency per US dollar series are used. Consumption and GDP data are seasonally adjusted, and exchange rates are period average series. From Balance of Payments (BOP), asset and liability flows (direct investment, portfolio investment, other investment, and reserves) are used. Data frequency is quarterly. Sample is a group of advanced countries from 1980Q1 to 2019Q1 for the gross flows and 2022q1 (or the latest available quarter, depending on countries) for the IFS variables. I follow the IFS definition of advanced economies.[35]

Measurement of risk sharing Using the real GDP and final consumption panel data, I run the following OLS regression as in Bai and Zhang (2012).

$$\Delta \ln c_t^j - \Delta \ln \bar{c}_t = \beta_0 + \beta_1(\Delta \ln y_t^j - \Delta \ln \bar{y}_t) + \varepsilon_t^j$$

where c_t^j is the real final consumption of the country j in time t, and y_t^j is the corresponding real GDP. $\Delta \ln x_t^j$ is a difference of log variables x in t and $t-4$, so that the differences represent annual growth rates of a variable x. $\Delta \ln \bar{x}_t$ is a simple mean of $\Delta \ln x_t^j$ across countries, so that the dependent and independent variables only contain cross-country differences within a quarter.

The main coefficient of interest is β_1, which is expected to be 0 if countries can insure themselves perfectly against idiosyncratic shocks on output. (See Bai and Zhang, 2012 for a more comprehensive review of the regression design.) Therefore, as β_1 gets closer to 0, it implies a higher level of international risk sharing. In the following, I construct a variable $risk\_sharing$, which is equal to $1-\beta_1$, so that a higher value of $risk\_sharing$ indeed implies more cross-country insurance.

Using a panel of real consumption and GDP variables, I perform a rolling-based regression

[35] The list of advanced countries include: Andorra, Principality of, Australia, Austria, Belgium, Canada, China, P.R.: Hong Kong, China, P.R.: Macao, Cyprus, Czech Rep., Denmark, Estonia, Rep. of, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea, Rep. of, Latvia, Lithuania, Luxembourg, Malta, Netherlands, The, New Zealand, Norway, Portugal, San Marino, Rep. of, Singapore, Slovak Rep., Slovenia, Rep. of, Spain, Sweden, Switzerland, Taiwan Province of China, United Kingdom, United States.

with a window of 40 quarters, starting from 1980Q1 to 2019Q1. As the sample ends around 2022Q1, towards the end of the sample period the number of observations attenuates. Note that the risk sharing estimation in 1980Q1, for example, includes the observations from 1980Q1 to 1989Q4.

Total gross flows to GDP The aggregate measure of gross flows to GDP used in the analysis is constructed as a fraction of aggregate gross flows over aggregate GDP among advanced countries. To be more specific, the sum of gross flows is taken as a total of asset and liability flows from the Balance of Payments (direct investment, portfolio investment, other investment, and reserves) from sample countries in each quarter.

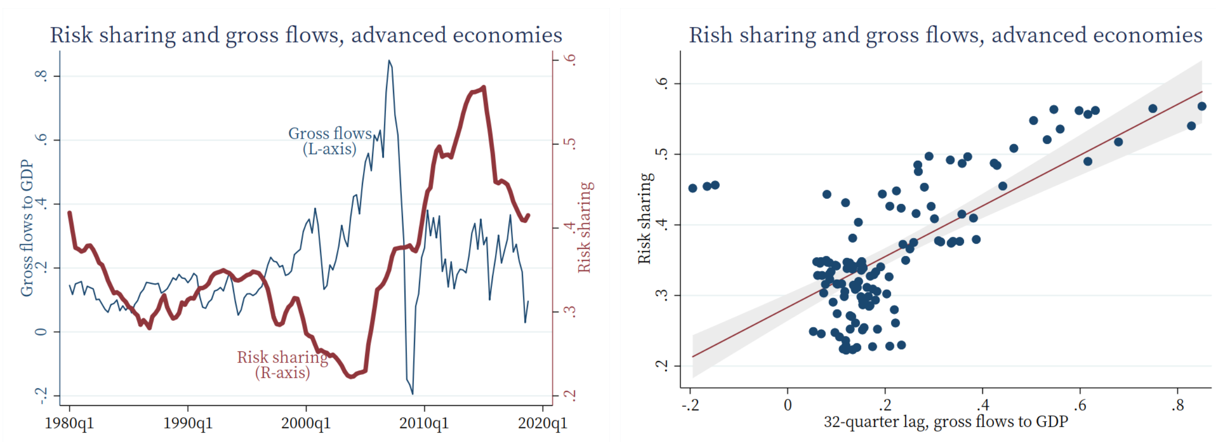

Figure 13: Time series of measured risk sharing and gross flows

Note: Gross flows to GDP series is a 3-quarter rolling average with equal weights to the previous, current, and future quarters.

Risk sharing and gross flows Figure 13 summarizes the time series of *risk_sharing* estimation and the total gross flows to GDP. On the left panel, it describes that the fraction of gross flows to GDP among advanced economies follows a very similar pattern as in the US: an explosive growth in the run-up to the Great Recession, followed by a sharp collapse and slow recovery. The risk-sharing shows a similar time series, only with some lags. The peak of measured risk sharing, for example, occurs in 2015Q1, 8 years after the peak of gross flows to GDP in 2007Q1. Comparing the time series of risk sharing with a 32-quarter lagged series of gross flows, as plotted on the right panel, there is a strong positive correlation between the *risk_sharing* and gross flows to GDP ratio with a coefficient of 0.1.

B Collateral constraints and default renegotiation

Before the beginning of each period, which is prior to the realization of new shocks and repayment of bonds, borrowers decide whether to default or not. If a person defaults, then creditors have rights to seize the Lucas Tree i that borrower in country i owns ($\theta^i(s)$) for the current state s. Assume that at the time of liquidation, the market value of Tree is zero with some probability, in the spirit of Jermann and Quadrini (2012).

With probability $\chi(s)$, the value of Tree i is $p^i_{Ti}(s)Q_i(s)$ when liquidated (in units of country i final goods), where i is the country of borrower. With probability $1-\chi(s)$, the Tree does not have any market value. Both parties enter the renegotiation process in case of a default, and I assume that the borrower has all the renegotiation power. In the following, I derive a collateral constraint where the borrower is indifferent from defaulting on the bond and keeping the promise. Without loss of generality, the case of a borrower in country 1 is described.

First, surplus of renegotiation on the borrower's side when the value of Tree is $p^i_{Ti}(s)Q_i(s)$:

$$\sum_{s'} \pi(s'|s)V_1(w(s');s',W(s')) - a_1^{1\prime}(s)\mathbb{1}_{(a_1^{1\prime}(s)<0)} - e(s)a_2^{1\prime}(s)\mathbb{1}_{(a_2^{1\prime}(s)<0)} - p^1_{T1}(s)Q_1(s)\theta^1(s) \quad (19)$$

In case that the Tree has zero market value, renegotiation surplus is:

$$\sum_{s'} \pi(s'|s)V_1(w(s');s',W(s')) - a_1^1(s)\mathbb{1}_{(a_1^1(s)<0)} - e(s)a_2^1(s)\mathbb{1}_{(a_2^1(s)<0)} \quad (20)$$

Then, in expectation, renegotiation value for borrower is:

$$\sum_{s'} \pi(s'|s)V_1(w(s');s',W(s')) - a_1^1(s)\mathbb{1}_{(a_1^1(s)<0)} - e(s)a_2^1(s)\mathbb{1}_{(a_2^1(s)<0)} - \chi(s)p^1_{T1}(s)Q_1(s)\theta^1(s) \quad (21)$$

For the borrower to be indifferent between defaulting and repaying the debt, the value of non defaulting should be at least as big as expected renegotiation value.

$$\begin{aligned}&\sum_{s'}\pi(s'|s)V_1(w(s');s',W(s')) \geq \\ &\quad \sum_{s'} \pi(s'|s)V_1(w(s');s',W(s')) - a_1^1(s)\mathbb{1}_{(a_1^1(s)<0)} - e(s)a_2^1(s)\mathbb{1}_{(a_2^1(s)<0)} - \chi(s)p^1_{T1}(s)Q_1(s)\theta^1(s)\end{aligned} \quad (22)$$

Above inequality is equal to the collateral constraint after rearrangement.

C Proofs of complete markets: risk sharing intuition

Rewriting the budget constraint for the household 1, for any states $(s, s') \in \{s_1, s_2\}$,

$$\sum_{i=1}^{2} p_i^1(s')c_i^1(s') + \begin{bmatrix} q_1(s') & e(s')q_2(s') \end{bmatrix} \begin{bmatrix} a_1^{1\prime}(s') \\ a_2^{1\prime}(s') \end{bmatrix} = p_1^1(s')y_1(s') + \begin{bmatrix} 1 & e(s') \end{bmatrix} \begin{bmatrix} a_1^1(s) \\ a_2^1(s) \end{bmatrix} \tag{23}$$

Simplifying notations:

$$A(s) \equiv \begin{bmatrix} 1 & e(s) \end{bmatrix} \tag{24}$$

$$B(s) \equiv \begin{bmatrix} \sum_{i=1}^{2} p_i^1(s)c_i^1(s) - p_1^1(s)y_1(s) \end{bmatrix} \tag{25}$$

$$C(s) \equiv \begin{bmatrix} q_1(s) & e(s)q_2(s) \end{bmatrix} \tag{26}$$

$$X(s) \equiv \begin{bmatrix} a_1^1(s) & a_2^1(s) \end{bmatrix}' \tag{27}$$

Then, 2 (today's states) $\times$ 2 (tomorrow's states) = 4 budget constraints are:

$$A(s')X(s) = B(s') + C(s')X(s'),\ \forall s', s \in \{s_1, s_2\} \tag{28}$$

The probability of each state is given as half, independent of time.

$$\pi(s_1) = \pi(s_2) = 0.5 \tag{29}$$

Conjecture that $X(s_1) = X(s_2) = X^*$. Then, above equation is

$$A(s_j)X^* = B(s_j) + C(s_j)X^* \tag{30}$$

Define a matrix of coefficients for all states.

$$A = \begin{bmatrix} A(s_1) \\ A(s_2) \end{bmatrix},\ B = \begin{bmatrix} B(s_1) \\ B(s_2) \end{bmatrix},\ C = \begin{bmatrix} C(s_1) \\ C(s_2) \end{bmatrix} \tag{31}$$

$$AX^* = B + CX^* \tag{32}$$

Then, if $(A - C)$ is invertible, we can solve for X^*.

$$X^* = (A - C)^{-1}B \tag{33}$$

Now we use the social planner's allocations to characterize this bond position. First order necessary condition of social planner equalizes marginal utility of tradable i consumption between two countries, $\forall i = 1, 2$.

$$u_i^1(c^{1^*}(s)) = u_i^2(c^{2^*}(s)) \tag{34}$$

where $u_i^j(c^j(s))$ is a marginal utility of country j at state s w.r.t. good i. More explicitly,

$$u_1^1(c^1(s)) = c^1(s)^{-\gamma+1/\sigma}(\omega_H/c_1^1(s))^{1/\sigma} \tag{35}$$

$$u_1^2(c^2(s)) = c^2(s)^{-\gamma+1/\sigma}(\omega_F/c_1^2(s))^{1/\sigma} \tag{36}$$

Given that utility functions are symmetric across countries, social planner's FOC reduces to the following equations.

$$\left(\frac{c^1(s)}{c^2(s)}\right)^{-\sigma\gamma+1} = \frac{c_1^1(s)/\omega_H}{c_1^2(s)/\omega_F} = \frac{c_2^1(s)/\omega_F}{c_2^2(s)/\omega_H} \tag{37}$$

Moreover, due to symmetry, $u_1^1(s_1) = u_2^2(s_2)$. Then, plugging in the symmetry to the bond prices, define $q(H) \equiv q_1(s_1) = q_2(s_2)$ and $q(L) \equiv q_1(s_2) = q_2(s_1)$. Also, $e(H) \equiv e(s_1) = p_1^{1^*}(s_1)/p_1^{2^*}(s_1) = p_2^{2^*}(s_2)/p_2^{1^*}(s_2) = 1/e(s_2)$. Rewriting the matrix,

$$A - C = \begin{bmatrix} 1 - q_1(s_1) & e(s_1)(1 - q_2(s_1)) \\ 1 - q_1(s_2) & e(s_2)(1 - q_2(s_2)) \end{bmatrix} \tag{38}$$

$$= \begin{bmatrix} 1 - q(H) & (1 - q(L))e(H) \\ 1 - q(L) & (1 - q(H))/e(H) \end{bmatrix} \tag{39}$$

$$det(A - C) = (1 - q(H))^2/e(H) - (1 - q(L))^2 e(H) \tag{40}$$

If $det(A - C)$ is not 0, then $A - C$ is invertible.

Define net export of country 1 in state s_1 as $nx(H)$

$$nx(H) = -\sum_{i=1}^{2} p_i^{1^*}(s_1)c_i^{1^*}(s_1) + p_1^{1^*}(s_1)y_H \tag{41}$$

Then, asset positions are:

$$
\begin{aligned}
X^* &= (A-C)^{-1}B && (42)\\
&= \frac{1}{det(A-C)}\begin{bmatrix} 1-q(H) & -(1-q(L))e(H) \\ -(1-q(L)) & (1-q(H))/e(H) \end{bmatrix}\begin{bmatrix} \sum_{i=1}^{2} p_i^{1*}(s_1)c_i^{1*}(s_1) - p_1^{1*}(s_1)y_H \\ \sum_{i=1}^{2} p_i^{1*}(s_2)c_i^{1*}(s_2) - p_1^{1*}(s_2)y_L \end{bmatrix} && (43)\\
&= \frac{1}{(1-q(H))^2/e(H) - (1-q(L))^2 e(H)}\begin{bmatrix} 1-q(H) & -(1-q(L))e(H) \\ -(1-q(L)) & (1-q(H))/e(H) \end{bmatrix}\begin{bmatrix} -nx(H) \\ nx(H) \end{bmatrix} && (44)\\
&= \frac{nx(H)}{1-q(H)-(1-q(L))e(H)}\begin{bmatrix} -e(H) \\ 1 \end{bmatrix} && (45)
\end{aligned}
$$

The denominator $1-q(H)-(1-q(L))e(H)$ is excess returns of bond 1 to bond 2 in state s_1, in terms of country 1 final goods. The equation shows that if the net exports are bigger, and excess returns are smaller, the larger absolute value of positions.

Parameters for the numerical exercise Parameters for the complete markets model section (Section 3) are set to be equal to the values of the quantitative analysis part (Section 4), except for the output levels which are arbitrary.

Table 2: Parameters

Parameter	Description	Value
β	Discount factor	0.98
γ	Risk aversion	1.00
ω_H	Home bias	0.855
y_H	High output level	1.20
y_L	Low output level	0.80

D Numerical algorithm

In order to solve the model globally, I use the time iteration algorithm by Kubler and Schmedders (2003), which has been applied to other international portfolio choice models such as Stepanchuk and Tsyrennikov (2015) and Dou and Verdelhan (2015). The algorithm finds equilibrium policy functions starting from an initial guess, by solving a system of first order necessary conditions and Kuhn-Tucker conditions and updating guesses over iterations.

This equilibrium is 'ε-equilibrium', meaning that the policy functions are solved up to some given critical value $\varepsilon > 0$ accuracy. There are total of T time periods where collateral constraints evolve over time. Specifically, denote a set of endogenous variables at iteration k in time t as $\Omega(k;t) = \{w(k;t), c_i^j(k;t), a_i^j(k;t), e(k;t), p_i^j(k;t), q_i(k;t), Q_i(k;t), \xi^j(k;t)\},\ i,j = 1,2$. Here, I have used the equilibrium condition that aggregate endogenous variables are same as individual ones. Also, ξ^j is a Garcia-Zangwill parameter that solves for Kuhn-Tucker conditions in country j. Also define all endogenous variables except for net position $w(k;t)$ as $\tilde{\Omega}(k;t) = \Omega(k;t)/w(k;t)$, since $w(k;t)$ is an endogenous state variable. Functions that are arguments of set $\tilde{\Omega}$ have net wealth fraction and exogenous states as their input ($f : \mathbb{R}^4 \to \mathbb{R}^1,\ \forall f \in \tilde{\Omega}$), which are suppressed in the following expression. The algorithm proceeds as follows.

1. Set up the initial guesses of $\tilde{\Omega}(0;T)$ and grids for net position w.

   I set equispaced grids for net position with 51 points[36] for an interval [0.25,0.75], and set steady state prices for $q_j(0;T) = \beta$ and $Q_j(0;T) = 1/(1-\beta)$. I start with zero bond positions for all bonds in all countries, $a_i^j(0;T) = 0$. Finally, initial Garcia-Zangwill parameters $\xi_j^i(0;T)$ are set to $\sqrt{\chi Q_j(0;T)}$. Set $\tilde{\Omega}(0;T) = \tilde{\Omega}^o$, where $\tilde{\Omega}^o$ is a set of *old* policy functions that is updated in every iteration. Exogenous state variables are discretized to 3 points for each output shock using the method by Tauchen (1986), and critical value is set to $\varepsilon = 10^{-3}$. The initial guess for the steady state level collateral constraints $\chi(\zeta = 1;T) = \chi_o$ is set at a high enough level.

2. Start the time iteration in time t.

   For any iteration $k \geq 1$, given the previous iteration's guess as *future* endogenous policy functions and prices[37], solve a system of first-order conditions and Kuhn Tucker conditions at each grid point of state. I solve the system of equations at precision of 10^{-5}, using modified Powell's non linear solver[38]. Using the solutions in iteration k, update functions $f^o \in \tilde{\Omega}^o$ as a convex combination of $f(k;t) \in \tilde{\Omega}(k;t)$ and f^o: $f^o = \delta f(k;t) + (1-\delta) f^o$.

[36] For the sensitivity analysis, 23 grid points

[37] Here, I need to find the mapping of today's net wealth fraction $w(s_t)$ to the tomorrow's net wealth fraction $w(s_{t+1})$ for any future state s_{t+1} by finding a root in equation 9. Since the solution often lies off of the grid points, I use spline methods to interpolate policy functions across endogenous state grid of $w(s_{t+1})$. I used B-spline method by Habermann and Kindermann (2007).

[38] I use HYBRD1 in Minpack. When there are points that cannot be solved, I impute the solution. If there are nearby neighboring points that are solved correctly, I use the linear interpolation at the initial stage and spline interpolation at the later stage. For the regions where the collateral constraint is binding, I interpolate the equity prices and guess the asset holding policy functions.

3. The algorithm stops in time t when maximum absolute difference of policy, price, and Garcia-Zangwill parameters between k^{th} iteration and *old* function across all state grid is less than critical value ε, $\max_{(w,s)} ||f(k;t) - f^o|| < \varepsilon, \ \forall f \in \tilde{\Omega}$, or $k > K_{max}$. Check whether the collateral constraint is non-binding at the zero-shock state for $t = T$ and whether the collateral constraint is binding at the zero-shock state for $t < T$. If not, adjust the initial guess and go back to step 1. Otherwise, move to the next step.

4. Repeat steps 2-4 until $t = 1$. The algorithm stops.

E Extension to an economy with non-tradable goods

While the benchmark economy abstracts from non-tradable goods, a more general framework featuring both non-tradable and tradable goods can carry similar institutions as in the tradable-only model. In this section, a model setup with non-tradable goods and complete market results are presented. All parts not mentioned in this appendix are identical or analogous to the benchmark setup.

Household utility Household utility functions are analogous to the benchmark model. The difference is that the aggregate tradable good and non-tradable good are combined to an aggregate consumption basket:

$$c^1 = \left(\omega_N^{\frac{1}{\sigma}} \left(c_N^1\right)^{\frac{\sigma-1}{\sigma}} + \omega_T^{\frac{1}{\sigma}} \left(c_T^1\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{46}$$

where σ is the elasticity of substitution between tradable and non-tradable, and weights on non-tradable and tradable goods are ω_N and $\omega_T = 1 - \omega_N$, respectively.

Household budget constraint Compared to the benchmark model, consumption and endowment of the non-tradable goods are added. However, notice that since non-tradable goods are consumed only by the domestic households, the non-tradable consumption and endowments cancel out in equilibrium.

$$\begin{aligned} \sum_{i=1}^{2} p_{Ti}^1(s^t)c_{Ti}^1(s^t) + p_N^1(s^t)c_N^1(s^t) + q_1(s^t)a_1^1(s^t) + e(s^t)q_2(s^t)a_2^1(s^t) + p_{T1}^1(s^t)Q_1(s^t)\theta^1(s^t) \\ \leq a_1^1(s^{t-1}) + e(s^t)a_2^1(s^{t-1}) + p_{T1}^1(s^t)\left(y_{T1}(s^t) + Q_1(s^t)\right)\theta^1(s^{t-1}) + p_N^1(s^t)y_N^1(s^t) \end{aligned} \tag{47}$$

Notice that the final good in country 1 is used as a numeraire.[39]

Market clearing Goods markets clear for each state, and bonds have zero net supply. Equities are owned by domestic households only.

$$\sum_{j=1}^{2} c_{Ti}^{j}(s^t) = y_{Ti}(s^t),\ c_N^j(s^t) = y_N^j(s^t)\ ,\sum_{j=1}^{2} a_i^j(s^t) = 0,\ \theta^j(s^t) = 1,\ \forall s^t \in S^t,\ i,j = 1,2 \quad (48)$$

Net wealth fraction and recursive formulation Net wealth fraction is defined analogously as in the benchmark model.

$$V_1\left(w(s), \theta^1; W(s), s\right) = \max_{c_{Ti}^1, c_N^1, a_i^{1\prime}, \theta^{1\prime}} u(c^1) + \beta \sum_{s'} \pi(s'|s) V_1\left(w(s'); W(s'), s'\right) \quad (49)$$

$$\begin{aligned}
s.t.\ \ & \sum_{i=1}^{2} p_{Ti}^1\left(W(s), s\right) c_{Ti}^1 + p_N^1\left(W(s), s\right) c_N^1 + q_1\left(W(s), s\right) a_1^{1\prime} + e\left(W(s), s\right) q_2\left(W(s), s\right) a_2^{1\prime} \\
& \leq w(s) \left[p_{T1}^1\left(W(s), s\right) y_1(s) + p_{T2}^1\left(W(s), s\right) y_2(s)\right] \\
& \quad + p_N^1\left(W(s), s\right) y_N^1(s) + p_{T1}^1(W(s), s) Q_1(W(s), s)(\theta^1 - \theta^{1\prime}) \qquad (50) \\
& a_1^{1\prime} \mathbb{1}_{\left(a_1^{1\prime} < 0\right)} + e\left(W(s), s\right) a_2^{1\prime} \mathbb{1}_{\left(a_2^{1\prime} < 0\right)} \geq -\chi^1(s) p_{T1}^1\left(W(s), s\right) Q_1\left(W(s), s\right) \theta^1 \qquad (51) \\
& W(s') = \Gamma\left(W(s), s'; s\right),\ \forall s' \in S \qquad (52) \\
& w\left(s'; W(s'), a_i^{1\prime}, \theta^{1\prime}\right) = \frac{p_{T1}^1\left(W(s'), s'\right) y_{T1}(s') \theta^{1\prime} + a_1^{1\prime} + e\left(W(s'), s'\right) a_2^{1\prime}}{p_{T1}^1\left(W(s'), s'\right) y_{T1}(s') + e\left(W(s'), s'\right) p_{T2}^2\left(W(s'), s'\right) y_{T2}(s')} \qquad (53)
\end{aligned}$$

Complete markets Complete markets setting is analogous to the benchmark case. The difference is that now non-tradable endowment moves together with the tradable output. In summary, $s_1 = (y_{T1} = y_N^1 = y_H, y_{T2} = y_N^2 = y_L)$, $s_2 = (y_{T1} = y_N^1 = y_L, y_{T2} = y_N^2 = y_H)$. In every time period, there is an equal probability of 0.5 that each state realizes, i.i.d. The social planner maximizes the sum of two countries' flow utilities with equal weights, subject to feasibility constraints of each state.

$$U^*(s) = \max_{\{c_{Ti}^j, c_N^j\}, i,j=1,2} u_1\left(c^1(s)\right) + u_2\left(c^2(s)\right) \quad (54)$$

$$s.t.\ c_{Ti}^1(s) + c_{Ti}^2(s) = y_{Ti}(s),\ c_N^j(s) = y_N^j,\quad i = 1,2 \quad (55)$$

[39]In particular, prices are aggregated and normalized as in the following:

$$p_T^1 = \left(\omega_H (p_{T1})^{1-\sigma_T} + \omega_F (p_{T2})^{1-\sigma_T}\right)^{1/(1-\sigma_T)}$$
$$P_1 = \left(\omega_T (p_T^1)^{1-\sigma} + \omega_N (p_N^1)^{1-\sigma}\right)^{1/(1-\sigma)}$$

where $P_1 = 1$ as a normalization. Prices in country 2 are set analogously.

All propositions and corollaries are analogous to the benchmark model.

Proposition 2. *If excess returns of bond 2 to bond 1 in state 1 (s_1) is not zero, then there is a unique bond portfolio $a^{1*} = (a_1^{1*}, a_2^{1*})'$ that decentralizes the social planner's allocations. Specifically,*

$$a^{1*} = \frac{nx_1^*}{rx_1^*} \begin{bmatrix} e^*(s_1) \\ -1 \end{bmatrix} \tag{56}$$

where nx_1^* and and rx_1^* are the first best net exports and excess returns of bond 2 to bond 1, respectively, in country 1 and state 1 (s_1).

$$nx_1^* = {p_{T1}^1}^*(s_1)[y_H - {c_{T1}^1}^*(s_1)] - {p_{T2}^1}^*(s_1){c_{T2}^1}^*(s_1)$$

$$rx_1^* = (1 - q_2^*(s_1))\, e(s_1) - (1 - q_1^*(s_1))$$

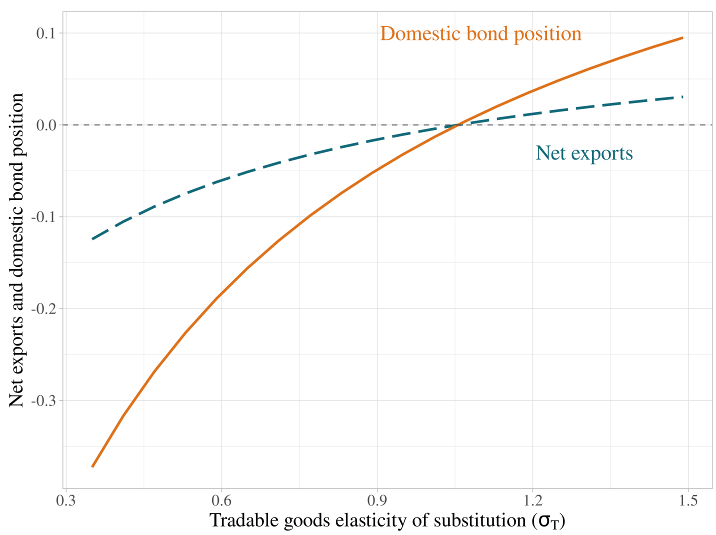

Figure 14: Net exports (nx_1^*) and domestic bond positions (a_1^{1*}) on σ_T

Note: All figures are for country 1, state 1 (s_1).

Corollary 3. *Given an elasticity of substitution between two tradable goods σ_T and all other parameters ρ, define first best net exports of country 1 in state 1 (s_1) as*

$$nx(\sigma_T; \rho) \equiv {p_{T1}^1}^*(\sigma_T; \rho)[y_H - {c_{T1}^1}^*(\sigma_T; \rho)] - {p_{T2}^1}^*(\sigma_T; \rho){c_{T2}^1}^*(\sigma_T; \rho)$$

where x^* denotes the social planner's solution in s_1 for any variable x. Then, $sign(a_1^{1*}) = sign(nx(\sigma_T; \rho))$.

Corollary 4. *Let σ_T^* such that $nx(\sigma_T^*;\rho)=0$. Then, for any $\sigma_T<\sigma_T^*$, ${a_1^1}^*<0$.*

An analogous numerical exercise shows that the same relationship between tradable goods elasticity of substitution and portfolio positions hold, as Figure 14 shows. Proof is omitted as it is similar to the benchmark case.

F Extension to an economy with trade frictions

As an extension of the benchmark economy, I include trade frictions (τ) in an otherwise identical environment. Then, the budget constraint of the Home household becomes:

$$\begin{aligned} p_1^1(s^t)c_1^1(s^t) + \frac{p_2^1(s^t)}{1-\tau}c_2^1(s^t) + q_1(s^t)a_1^1(s^t) + e(s^t)q_2(s^t)a_2^1(s^t) + p_1^1(s^t)Q_1(s^t)\theta^1(s^t) \\ \leq a_1^1(s^{t-1}) + e(s^t)a_2^1(s^{t-1}) + p_1^1(s^t)\left(y_1(s^t)+Q_1(s^t)\right)\theta^1(s^{t-1}) + T^1(s^t) \end{aligned}$$

where a positive value of trade friction τ, as a form of tariff, is applied only to the Foreign good price p_2^1. The tariff revenue is redistributed in each period as a lump-sum subsidy T^1. Therefore, in equilibrium, $T^1 = (\frac{1}{1-\tau}-1)p_2^1c_2^1 = \frac{\tau}{1-\tau}p_2^1c_2^1$, and analogously for the Foreign country. Under this setting, with a τ of 1%, the equilibrium asset and liability holdings decrease by 1.35% at the zero-shock state compared to the benchmark. This shows that an increase of goods trade frictions leads to a decreased level of international financial transactions, which implies a lower degree of risk sharing across borders.

G Sensitivity analysis calibration table

Table 3: Parameters: Sensitivity Analysis

Parameter	Description	Value
High σ		
χ_{ss}	Steady state collateral constraint	0.0193
Low σ		
χ_{ss}	Steady state collateral constraint	0.080
High risk aversion		
χ_{ss}	Steady state collateral constraint	0.021

H Data: Calibration targets

- Source: OECD Statistics, Gross domestic product (expenditure approach: current prices, domestic currency & volume index), exchange rates (period average, national currency per US dollar)

- Rest of the World (RoW) dataset is constructed using an unbalanced panel of available countries. Weight is constructed as a fraction of world GDP in USD in each year. World GDP in USD is a sum of available current national currency GDP over USD exchange rate in each year.

- List of countries (data start year - data end year): Argentina (2004 - 2019) , Australia (1970 - 2019) , Austria (1970 - 2019) , Belgium (1970 - 2019) , Brazil (1995 - 2017) , Bulgaria (1995 - 2019) , Canada (1970 - 2019) , Chile (1986 - 2019) , China (People's Republic of) (1970 - 2019) , Colombia (1975 - 2019) , Costa Rica (1991 - 2019) , Croatia (1995 - 2019) , Cyprus (1995 - 2019) , Czech Republic (1990 - 2019) , Denmark (1970 - 2019) , Estonia (1993 - 2019) , Finland (1970 - 2019) , France (1970 - 2019) , Germany (1970 - 2019) , Greece (1970 - 2019) , Hong Kong, China (1970 - 2017) , Hungary (1991 - 2019) , Iceland (1970 - 2019) , India (1970 - 2017) , Indonesia (1993 - 2019) , Ireland (1970 - 2019) , Israel (1970 - 2019) , Italy (1970 - 2019) , Japan (1970 - 2019) , Korea (1970 - 2019) , Latvia (1994 - 2019) , Lithuania (1995 - 2019) , Luxembourg (1970 - 2019) , Malta (1995 - 2019) , Mexico (1970 - 2019) , Morocco (2007 - 2017) , Netherlands (1970 - 2019) , New Zealand (1970 - 2019) , North Macedonia (2000 - 2018) , Norway (1970 - 2019) , Poland (1990 - 2019) , Portugal (1970 - 2019) , Romania (1995 - 2019) , Russia (1993 - 2019) , Saudi Arabia (1970 - 2019) , Serbia (1997 - 2018) , Slovak Republic (1993 - 2019) , Slovenia (1991 - 2019) , South Africa (1970 - 2019) , Spain (1970 - 2019) , Sweden (1970 - 2019) , Switzerland (1970 - 2019) , Turkey (1970 - 2019) , United Kingdom (1970 - 2019)

I Additional figures

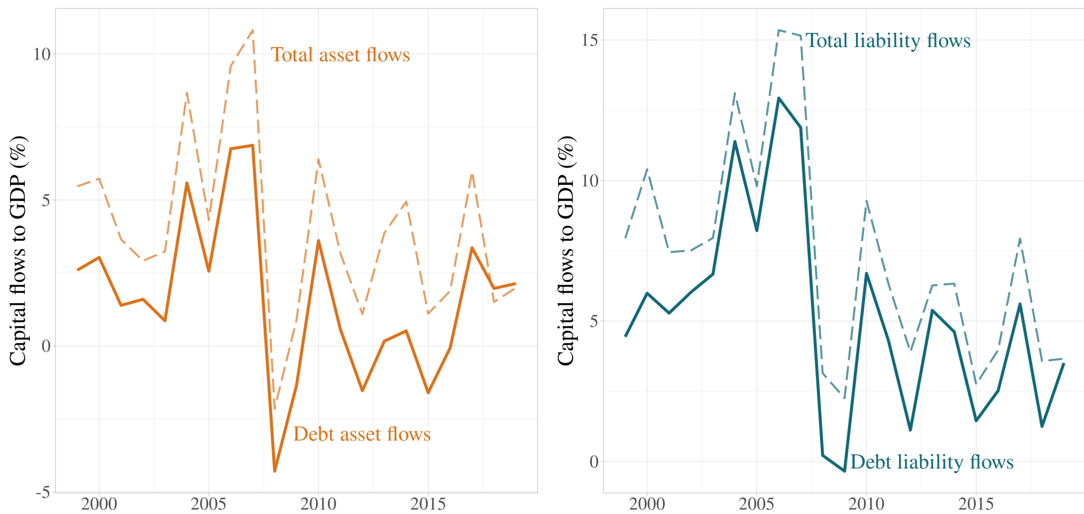

Figure 15: Liability and asset flows, total and debt flows, US against RoW

Note: Debt flows are calculated as the sum of Direct investment: Debt instruments, Portfolio investment: Debt securities, Other investment, and Reserve assets (only for debt asset flows, not for liabilities). Source: Bureau of Economic Analysis and author's calculations.

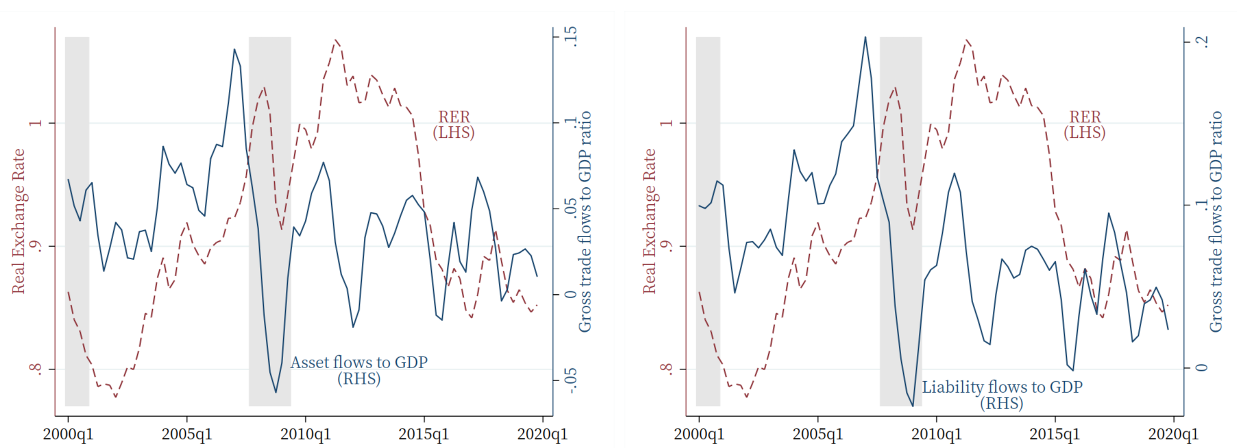

Figure 16: Asset and liability flows to GDP and real exchange rates

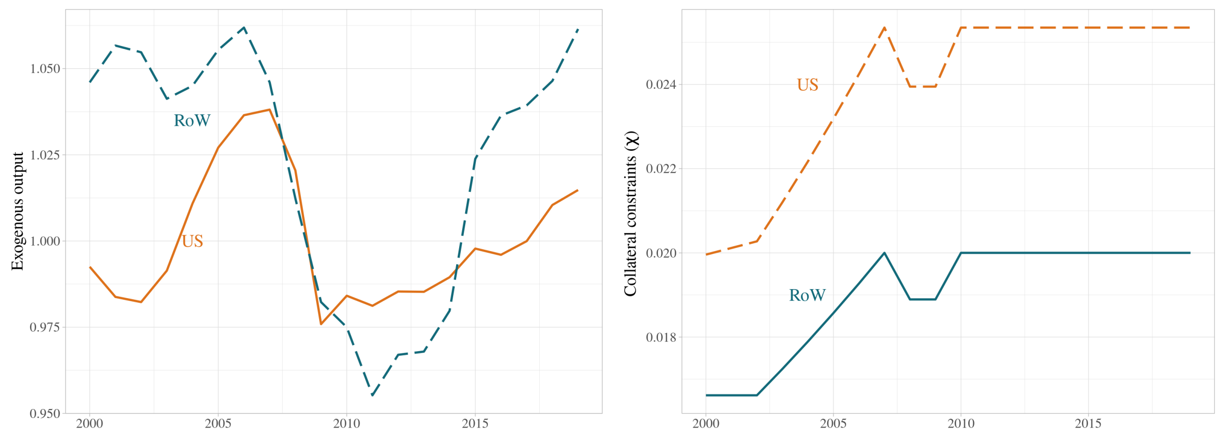

Figure 17: Exogenous output of the US and RoW (left) and collateral constraints of the US and RoW (right)

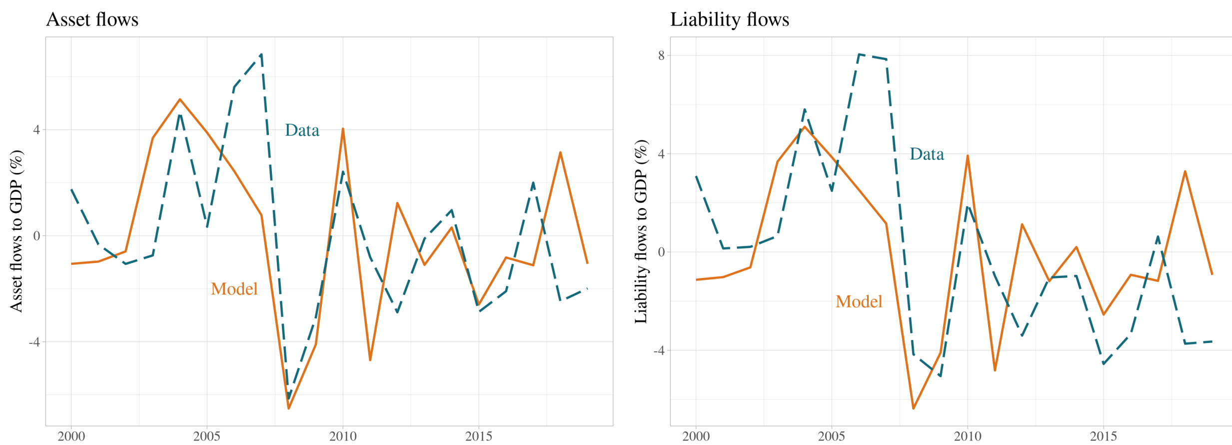

Figure 18: Asset and liability flows, data vs. model

Note: Both model simulated and data capital flows are de-meaned over the sample period.

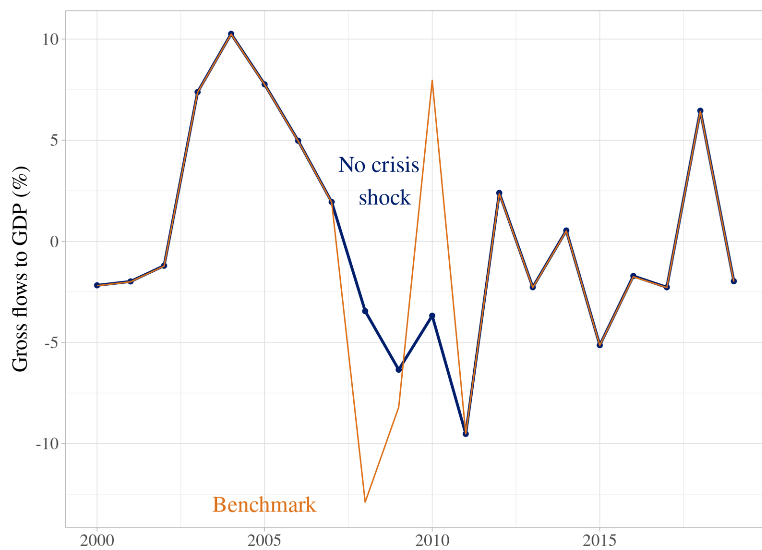

Figure 19: Counterfactual exercise: no crisis shock realization

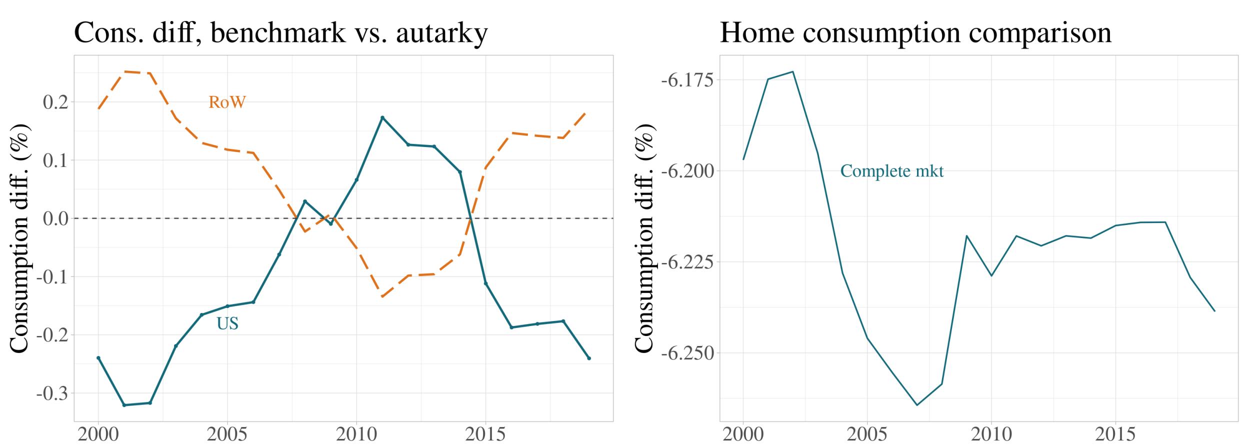

Figure 20: Home consumption differences by market completeness

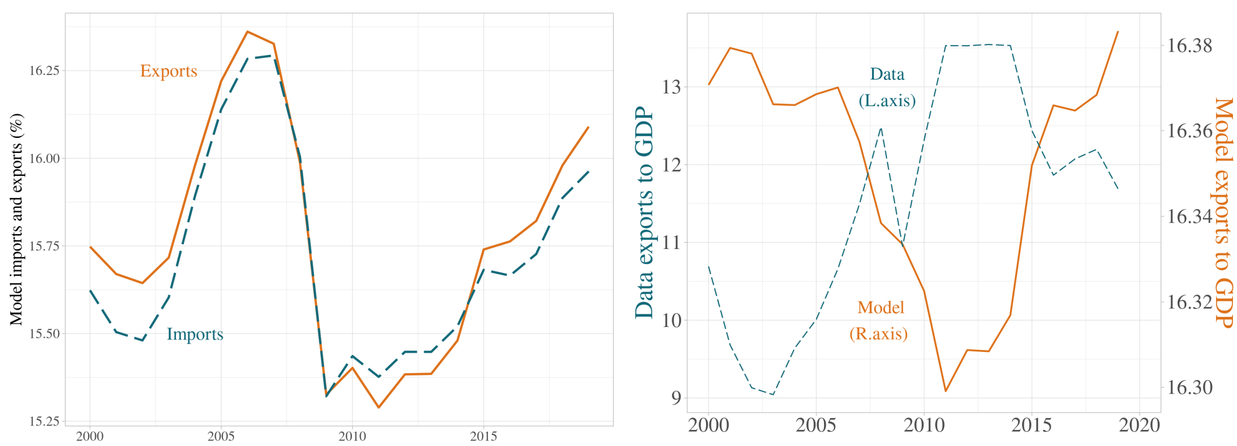

Figure 21: Imports and exports in the model