A Model of Gross Capital Flows:
Risk Sharing and Financial Frictions

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Abstract

This paper builds a two-country model of gross capital flows in which agents share tradable output risk using two bonds, subject to stochastic collateral constraints. Equilibrium portfolios are short in domestic bonds and long in foreign bonds because the endogenous movements of the real exchange rate provide a hedge against domestic output shocks. Under negative domestic shocks, these external positions transfer wealth from home to abroad. In an application to the Great Recession, the model shows that such wealth transfer from the US mitigated over 0.1 percentage point of the consumption drop abroad. Quantitatively, financial frictions can account for at least 55% of the collapse in gross flows of the US in 2008.

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Cross-border financial transactions have been growing steadily since the early 1980s, and then experienced a dramatic collapse in 2008. Gross flows, defined as the changes in international investment positions in both assets and liabilities, increased from 2% of GDP in 1970 to 26% in 2007 in the United States fueled by financial development and reduced capital controls. Subsequently, the gross flows dropped by 3.6 trillion dollars during the Great Recession, attaining the lowest level since 1980. The sudden collapse of gross flows during the financial turmoil opened a debate about whether gross flows matter, especially for developed countries, and about their implications for the real economy.

Classic studies of integrated financial markets have mainly focused on net flows, defined as the changes in international liability positions net of asset positions, or current accounts. Between 2007 and 2009, net flows fell by 3% of GDP in the United States, compared to a 23% drop in gross flows over the same period. This contrasts with the experience of emerging markets, which tend to have more volatile net flows and less volatile gross flows relative to developed countries (Broner et al. 2013). Motivated by these observations, I develop a theory of gross capital flows based on economic fundamentals and financial frictions, and evaluate the quantitative importance of these channels for integrated financial markets.

Accounting for gross flows is important because they affect the size of wealth transfers across countries, which are ultimately at the heart of consumption smoothing. One motivating observation during the Great Recession is that the sudden collapse of gross flows coincided with a sharp appreciation of the dollar and severe financial frictions. Because the United States mostly borrows in the dollar and invests in foreign currencies (Gourinchas and Rey 2007), a collapse in both inflows and outflows, combined with a simultaneous appreciation of the dollar, results in a limited wealth transfer from the United States to the Rest of the World. A natural question that arises is, how would the welfare of each economy change in the absence of gross flows? Also, how much of the collapse in gross flows was driven by financial frictions as opposed to economic fundamentals?

The main hypothesis is that households have a motive for sharing tradable output risk
which leads them to hold international investment positions, subject to collateral constraints. If domestic and foreign tradable goods are complements, it is optimal for households to import more foreign goods when domestic output is hit by a positive shock, and vice versa in response to a negative shock. Unless shocks to tradable output are perfectly correlated across countries, all households are better off by sharing output risk internationally via goods trading. In such cases, cross-border financial transactions are the instrument with which the risk sharing is implemented. The degree of risk sharing is bound by the value of the collateral.

To examine the mechanism of risk sharing through international financial transactions under financial frictions, this paper builds an open economy model of portfolio choice with two symmetric countries and two real bonds, where short positions are subject to stochastic collateral constraints. Each bond is denominated in the aggregate price index of the given country and promises to pay one unit of the aggregate consumption good next period. Each country produces one tradable good, whose output is exogenous and follows a stochastic process. In the presence of home bias in the aggregate consumption basket, real exchange rates (RER) are determined by relative prices of home to foreign goods. In this model, the RER plays a key role in the decision of investment positions for households, since it sets the returns on foreign bonds relative to domestic ones.

The equilibrium portfolio takes a short position (households borrow) in domestic bonds and a long position (households save) in foreign bonds. This is because the endogenous movements of the real exchange rate make this portfolio optimal for risk sharing. For example, when a positive shock hits tradable output in one country, its domestic tradable goods become less valuable relative to the foreign tradable goods, which implies real exchange rate depreciation. Combined with liabilities in domestic bonds and assets in foreign bonds, such depreciation of the RER results in an increase in the value of net international investment position (NIIP) for households. This increase in NIIP in turn can be used to purchase more foreign goods, which are complements to domestic goods in the calibrated model.

Adjustments to the equilibrium portfolio take place under two circumstances. First,
when the output of either country is hit by a shock, gross flows arise as a difference in gross positions. Negative shocks to output in any country result in lower positions for both domestic and foreign bonds because there is less risk to insure against. This implies that the model predicts a positive correlation of gross flows and output, just as observed in the data. Secondly, if the collateral constraints tighten, gross positions decrease and gross flows become negative. In the model, these two channels jointly drive the movements of gross flows.

To assess the quantitative power of these channels, I calibrate the model to data of the United States, which experienced a dramatic collapse in gross flows during the Great Recession. Using the data on gross output for years 2000-2019, I estimate the parameters of stochastic processes in the model and generate a number of testable implications regarding gross flows. Specifically, I feed in the paths of realized output in the US and the Rest of the World (RoW) and show that the model replicates the key stylized facts regarding gross flows. First, the simulated gross flows match the observed data closely, not only in terms of their magnitude and volatility, but also the patterns of gradual boom and sudden collapse. In addition, the model asset and liability flows are highly correlated and pro-cyclical, which is one of the key characteristics of gross flows in developed countries. Moreover, risk sharing and financial frictions can account for nearly all of the volatility in gross flows. In particular, the simulated results generate the collapse during the 2008 crisis to the same extent as in the data, while the financial friction channel alone can explain 55% of the decline. Importantly, this prediction is accompanied by a steep appreciation of the dollar, resulting in a reduced net external position of the US relative to RoW. This is in line with the exchange rate valuation effects documented by Lane and Shambaugh (2010).

The main result is that the observed fluctuations in gross flows mitigated a part of the consumption drop in the aftermath of the Great Recession in RoW, due to significant transfers of wealth from the US. Through the risk sharing mechanism of gross flows, the RoW consumption dropped by 3.01% from 2007 to 2011, which would have been 3.14% under the financial autarky. These gains in consumption for RoW came at the cost of losses in con-
sumption in the US, which instead enjoyed a higher level of consumption until 2007. Overall welfare in the world was improved through the risk sharing channel during the sample period. It is noteworthy that the magnitudes and directions of welfare gains depend on the levels of Armington elasticity. For example, a lower level enhances the welfare gains of RoW during the financial crisis.

The final application of my model is to analyze the effects of introducing a financial transaction tax. In 2011, the European Commission proposed a Financial Transaction Tax (FTT), which would levy a 0.1% tax on security transactions in order to reduce the volatility of asset markets and recently considered it again as a way to finance COVID-19 related expenditure. The calibrated model suggests that imposing the tax will result in reduced benefits of international diversification. I show that, through the lens of international risk sharing, the proposed tax on bond transactions will eliminate cross-border financial holdings and prevent households from sharing tradable consumption risk, resulting in lower welfare.

**Literature review** This paper is motivated by empirical observations on gross capital flows. Broner et al. (2013), Rey (2015), and Forbes and Warnock (2012) document large and volatile movements of gross flows, using an extensive panel of countries. More recently, Avdjiev et al. (2017) and Davis et al. (2021) analyze different types of gross flows and the contribution of global factors. This paper models gross flows that are not only consistent with the key characteristics documented in the literature, but also quantitatively matches the magnitude and volatility of gross flows in the data. A novel part of this paper is to quantitatively analyze the welfare implications of gross flows during the Great Recession, whose sudden collapse has drawn a significant amount of interest in the aforementioned literature. In order to do so, this paper is based on both classic international risk sharing and macro-finance literature with collateral constraints, and equipped with global solution methods for the endogenous portfolio choice.

This paper is closely related to an extensive literature that studies international diversification, which traditionally focuses on the long-run external positions (Stockman and Dallas
1989, Baxter et al. 1998, Heathcote and Perri 2013, among others). This paper is in line with the international diversification literature, where domestic households hold international positions in order to share their risk with foreign households. The departure point of this paper is to study the dynamics of international risk sharing over time, compared to a long-run international diversification that is not time varying. In order to do so, this paper builds an incomplete markets model where there are one-period uncontingent bonds and stochastic collateral constraints. This is not the first paper to study gross flows models. There is a growing literature that models gross flows, or more generally inflows and outflows separately (Tille and Van Wincoop, 2010, Dou and Verdelhan, 2015, Gourio et al., 2015, Davis and Van Wincoop, 2018, Caballero and Simsek, 2020). This paper contributes to the literature of gross flows by solving a quantitative model using global methods, which is specifically designed to study extreme events such as the Great Recession. In particular, the novel part of this paper is to incorporate occasionally binding collateral constraints that are widely used in macro-finance literature into the model of gross flows, and analyze the quantitative importance of this channel compared to the classic risk sharing based on fundamentals.

The literature of macro-finance with various financial frictions, and in particular collateral constraints, has seen an explosive growth especially since the financial crisis (Kiyotaki and Moore, 1997, Bernanke et al., 1999, Gertler and Kiyotaki, 2010). This paper shows that occasionally binding collateral constraints play a key role in gross flows, both in their gradual growth leading up to the financial crisis due to a loosening of constraints and a sudden collapse due to tightening constraints.

Finally, this paper develops a global solution method for gross flows. The global solution method was pioneered by Kubler and Schmedders (2003), and it departs from linear approximation models adopted by many existing papers on international capital flows. One of the closest models in international portfolio choice that this paper follows is Stepanchuk and Tsyrennikov (2015), where they focus on the consequences of debt market dominance rather than two-way capital flows as in this paper.
The rest of this paper is organized as follows. In Section 1, key characteristics of gross flows along with international investment positions are described. Section 2 studies a simple environment of complete markets with analytic solutions. Section 3 lays out the model environment. Section 4 brings the model to the data, calibrating for the US and the Rest of the World data from 2000 to 2019. Conclusion follows.

1 Data: Key characteristics of gross flows

In this section, the concept of gross flows and their key characteristics are described. The data observations on gross flows and characterizing statistics motivate the model design in the following section (Section 3). These characteristics are revisited in the Quantitative Analysis (Section 4), comparing the model results to the data.

Concept of gross flows  Gross capital flows are changes in international investment positions (IIP) due to transactions. IIP is a balance sheet of a country that records both assets, which are the financial claims on nonresidents (cross-border investments by domestic residents), and liabilities, which are the claims by nonresidents on residents (cross-border borrowings by domestic residents). Asset flows are changes in asset positions due to net acquisitions of foreign financial assets by domestic residents. Analogously, liability flows are changes in liability positions, which are equivalent to the net incurrence of liabilities by domestic residents. Gross capital flows are defined as the sum of asset and liability flows.

Boom, collapse, and slow recovery  Focusing on the sample period of 2000-2019 in the United States, I document some key characteristics of gross flows. First, asset and liability flows are highly correlated with the correlation coefficient of 0.94, as the left panel of Figure 1 demonstrates. This observation is consistent with the empirical literature, where Broner et al. (2013) document that the correlations have been increasing over time across 103 countries, especially after 2000. Davis and Van Wincoop (2018) also document increasing correlations
over the sample of 128 countries, and suggest that it is a result of increased financial and trade globalization. Second, gross flows are larger in levels, and much more volatile than net flows. As the right panel of Figure 1 depicts, gross flows peak 26% of GDP in 2007, and experience a sharp collapse to 1% of GDP in 2008. On the other hand, net flows, which are defined as liability flows net of asset flows, move from 4% to 5% during the same period, or 6% (2006) to 1% (2009) from peak to trough. While the net flows also have seen a significant change during the recession, the gross flows to GDP (7% std) are much more volatile than the net flows to GDP (1.5% std).

Both asset and liability flows are further decomposed into debt and equity flows (Lane and Milesi-Ferretti 2007), as Figure 2 summarizes in a diagram. Debt flows are composed of Portfolio investments in debt securities and Other investments, which are mostly banking flows. On the other hand, Direct investments and Portfolio investments in Equity and investment fund shares constitute equity flows. Debt gross flows are on average 66% of the

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1Financial derivatives, whose data is recorded in net positions only, and reserve assets, whose amount is small in the US, are not included in the classifications.
Motivated by the observation that debt flows take most of the gross flows, my model focuses only on bonds, and gross flows are calculated as the changes in bond holdings. Not only debt flows take the lion’s share in gross flows, the key characteristics described above for the total gross flows remain the same for the debt flows.

**Gross positions and currency composition** In order to compare model predictions on the equilibrium portfolio holdings with the data in the following quantitative section, I describe currency compositions of international investment positions in the United States. It is widely documented that the US hold mostly in dollars for its external liabilities and in foreign currencies for its external assets (Gourinchas and Rey 2007, Lane and Milesi-Ferretti 2007). Based on the data from Benetrix et al. (2019), the US holds 85% of external liabilities in dollars and 63% of external assets in foreign currency on average from 2000 to 2017. Later in the quantitative exercise (Section 4), the calibrated model predicts a short position in domestic bonds and a long position in foreign bonds simultaneously, which matches the well known currency composition of the US portfolio.

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2Since capital flows can take negative values, I calculate mean(abs(debt gross flows)/abs(total gross flows)) = 0.66.
2 Complete markets: risk sharing intuition

In this section, I analyze the risk sharing motive of bond portfolio choice under the complete markets setting before moving on to the full model with incomplete markets. I build a simple model where households choose their domestic and foreign bond positions in order to insure themselves against adverse shocks to output. I provide a closed form solution of the unique bond position, by which households achieve their first-best consumption allocations.

Physical environment There are two countries, country 1 and 2, where each country \( i = 1, 2 \) produces a single tradable good \( y_i \). Output processes are exogenous and follows a stochastic process with two possible states of High \( (y_H) \) or Low \( (y_L) \): \( y_i \in \{y_H, y_L\}, y_H > y_L \).

There are two symmetric states in the world economy, defined as \( s_1 \) and \( s_2 \), which belong to the set of all states \( S = \{s_1, s_2\} \). In state 1 \( (s_1) \), country 1’s output is High \( (y_1 = y_H) \) whereas country 2’s output is Low \( (y_2 = y_L) \). State 2 \( (s_2) \) is the symmetric case where the output is Low in country 1 and High in country 2: \( s_1 = (y_1 = y_H, y_2 = y_L), s_2 = (y_1 = y_L, y_2 = y_H) \).

In every time period, there is an equal probability of 0.5 that each state realizes, i.i.d.

Consumer utility There is a continuum of identical households in each country, whose measure is 1. Each household has a Constant Relative Risk Aversion (CRRA) utility with a Constant Elasticity of Substitution (CES) between two tradable goods. Formally,

\[
u(c^1(s)) = \frac{c^1(s)^{1-\gamma}}{1-\gamma}, \quad c^1(s) = (\omega_H^1 c^1_1(s))^{\frac{1}{\sigma}} + (\omega_F^1 c^1_2(s))^{\frac{1}{\sigma}} \quad (1)
\]

where \( \gamma \) is the risk aversion parameter. \( c^1(s) \) is the aggregate consumption basket of country 1 at state \( s \in S \), and \( c^1_1 \) and \( c^1_2 \) are the goods 1 and 2 consumption by country 1, respectively. Subscripts denote origins of goods and superscripts indicate destination of goods. \( \omega_H \) indicates weights on the home tradable good consumption, while \( \omega_F \), denoting weights on foreign good consumption, is set to \( 1 - \omega_H \). \( \sigma \) is the elasticity of substitution, or Armington elasticity, between the two goods. Utility of the country 2 is defined analogously.
Financial market  There are two one-period bonds, where each bond is denominated in the aggregate consumption basket of the given country. All bonds promise to pay one unit of aggregate consumption good uncontingent to the next period states. In the following, I denote $a^j_i(s)$ as the amount of bond $i$ purchased by households in country $j$ at state $s$. A negative value of $a^j_i$ implies borrowing, or short positions, in bond $i$ by households $j$.

Consumer’s problem  Individual households purchase goods and make portfolio decisions each period, under the following budget constraint at each state $s' \in S$ given the last period’s state as $s \in S$.

$$\sum_{i=1,2} p^1_i(s')c^1_i(s') + q_1(s')a^1_i(s') + e(s')q_2(s')a^2_i(s') \leq p^1_1(s')y_1(s') + a^1_1(s) + e(s')a^1_2(s) \quad (2)$$

where $a^1_i$ and $a^1_i'$ are the bond $i$ purchase from the previous period and the current period, respectively, and $q_i$ is the price of bond $i$. Notice that aggregate consumption basket in country 1 ($c^1$) is used as a numeraire.\(^3\) Here, $p^1_i$ is the price of consumption good $i$ in country 1 relative to the aggregate consumption basket, and $e$ is the exchange rate (price of country 2 final good in units of country 1 final good). Law of one price holds across countries. Consumers maximize their expected utility at time 0 under the budget constraints.

$$\max_{c^1(s^t),c^2(s^t),a^1(s^t),a^2(s^t)} \sum_{t=0}^{\infty} \sum_{s' \in S^t} \beta^t \pi(s^t)u(c^1(s')) \quad (3)$$

s.t.  $\sum_{i=1,2} p^1_i(s')c^1_i(s^t) + q_1(s')a^1_i(s^t) + e(s')q_2(s')a^2_i(s^t) \leq p^1_1(s')y_1(s^t) + a^1_1(s^{t-1}) + e(s')a^1_2(s^{t-1})$

Here, $s^t$ is the history of states from time 0 to time $t$, $s^t = (s_0, s_1, \ldots, s_t)$, where $s_k$ is the state at time $k$. $\pi(s^t)$ is time-0 probability of history $s^t$ realization.

\(^3\)Hence, price index of country 1 $P^1 = \left(\omega_H \left( p^1_1 \right)^{1-\sigma} + \omega_F \left( p^2_2 \right)^{1-\sigma} \right)^{1/(1-\sigma)}$ is normalized to be 1
Market clearing  Total demand of each goods are equal to the output.

\[ c_1^i(s) + c_2^i(s) = y_i(s), \ i = 1, 2 \] (4)

For the financial markets, there is a zero net supply of each bond for all states.

\[ a_1^i(s) + a_2^i(s) = 0, \ i = 1, 2 \] (5)

Social planner’s problem  Social planner maximizes the sum of two countries’ flow utilities with equal weights, subject to feasibility constraint of each state.

\[ U^*(s) = \max_{\{c_i^j\}, i, j = 1, 2} u_1(c_1^i(s)) + u_2(c_2^i(s)) \] (6)

s.t. \[ c_1^i(s) + c_2^i(s) = y_i(s), \ i = 1, 2 \] (7)

Complete market solution  I first solve for the social planner’s allocations, and then find the bond portfolio that decentralizes the first-best allocations. First order necessary conditions of the social planner characterize the optimal tradable consumption across households at each state. These allocations critically depend on the risk aversion and elasticity of substitution between home and foreign tradable goods. Following propositions describe the conditions that determine bond portfolios \( \{a_1^i\}, \ i, j = 1, 2 \). All proofs are in the Appendix.

Proposition 1. If excess returns of bond 2 to bond 1 in state 1 \( (s_1) \) is not zero, then there is a unique bond portfolio \( a^{1*} = (a_1^{1*}, a_2^{1*})' \) that decentralizes the social planner’s allocations. Specifically,

\[ a^{1*} = \frac{nx_1^*}{rx_1^*} \begin{bmatrix} e^*(s_1) \\ -1 \end{bmatrix} \] (8)

where \( nx_1^* \) and \( rx_1^* \) are the first best net exports and excess returns of bond 2 to bond 1,
respectively, in country 1 and state 1 \((s_1)\).

\[
nx_1^* = p_1^{1*}(s_1)[y_H - c_1^{1*}(s_1)] - p_2^{1*}(s_1)c_2^{1*}(s_1)
\]

\[
rx_1^* = (1 - q^*_2(s_1))c(s_1) - (1 - q^*_1(s_1))
\]

**Corollary 1.** Given an elasticity of substitution \(\sigma\) and all other parameters \(\rho\), define first best net exports of country 1 in state 1 \((s_1)\) as

\[
nx(\sigma; \rho) \equiv p_1^{1*}(\sigma; \rho)[y_H - c_1^{1*}(\sigma; \rho)] - p_2^{1*}(\sigma; \rho)c_2^{1*}(\sigma; \rho)
\]

where \(x^*\) denotes the social planner’s solution in \(s_1\) for any variable \(x\). Then, \(\text{sign}(a_1^{1*}) = \text{sign}(nx(\sigma; \rho))\).

**Corollary 2.** Let \(\sigma^*\) such that \(nx(\sigma^*; \rho) = 0\). Then, for any \(\sigma < \sigma^*\), \(a_1^{1*} < 0\).

If two tradable goods are complements with sufficiently low \(\sigma\), then the demand for foreign goods increases in the event of high domestic output. As Figure 3 describes for an example economy, below a certain level of \(\sigma\) country 1 becomes a net importer in the state of high domestic output \((s_1)\) under the social planner’s solution. In order to support such allocation of consumption goods, country 1 households must take a short (negative) position in their domestic bonds (bond 1, \(a_1^{1'}\)) while taking a symmetric long (positive) position in the foreign bonds (bond 2, \(a_2^{1'}\)). In the event that foreign output is lower than the domestic one, returns on foreign bonds (bond 2) are higher than domestic ones (bond 1). In other words, excess returns \((rx_1^*)\) on foreign bonds are positive in state 1. Therefore, by taking a long position in foreign bonds and simultaneously a short position in domestic bonds, households can finance their net imports in the event of high domestic output. Proposition 1 summarizes this relation between net exports, excess returns, and bond positions in a single equation. The size of bond positions are proportional to the size of net exports, and inversely related to the excess returns on foreign bonds relative to domestic bonds.
Finally, notice that there are no adjustments in the portfolio because there is a unique bond position that is common to all states. Therefore, there are no gross flows in the complete market case. In the following section, I set up the full model with incomplete financial markets where households adjust their portfolio in each state and hence gross flows arise.

3 The Model

In this section, I describe the physical environment of the full model and financial market structure. Following international portfolio choice models such as Baxter et al. (1998) and Tille and Van Wincoop (2010), I model an exchange economy where each country owns a Lucas Tree (Lucas, 1978) that produces a tradable good that follows stochastic processes. The key intuition of risk sharing from the previous section 2 holds in the main model, but financial markets are incomplete due to stochastic collateral constraints and noncontingent assets. Therefore, each household optimally adjusts the amount of asset holdings in each state and hence international capital flows are generated. This section studies the core mechanism of gross capital flows based on the risk sharing intuition derived in the earlier section and

Figure 3: Net exports \((nx_1^*)\) and domestic bond positions \((a_1^*)\) on \(\sigma\)

Note: All figures are for country 1, state 1 \((s_1)\). Parameter values are specified in Appendix.
financial frictions.

**Physical environment** In each period of time \( t = 0, 1, \ldots, \infty \), an exogenous state denoted as \( s_t \in S \) realizes. I denote the history of states from time 0 to time \( t \) as \( s^t = (s_0, s_1, \ldots, s_t) \), which is also called as a node in the event tree. The root of the event tree is given as \( s_0 \). The probability of a node \( s^t \) realization is denoted as \( \pi(s^t) \) in terms of time-0 probability, and the chance of node \( s^{t+1} \) realization given the history \( s^t \) is denoted as \( \pi(s^{t+1}|s^t) \). Events follow a Markov process, which is specified in the following paragraph.

In the model, there are two countries, 1 and 2, which are populated by a continuum of identical households with measure 1 in each country. Throughout this section, I focus on the problems of country 1 since the settings are symmetric across countries. There are two tradable goods \( y = (y_1, y_2) \) in the world, whose outputs are given as an exogenous process. Based on Lucas (1978), I assume that each country \( i = 1, 2 \) owns a Lucas Tree that produces tradable good output \( y_i \). The world output \( y(s^t) \) follows a log normal AR1 process, with mean \( \rho \log y(s^{t-1}) \) and covariance \( \Sigma \). To summarize,

\[
\begin{bmatrix}
\log y_1(s^t) \\
\log y_2(s^t)
\end{bmatrix} = \rho \begin{bmatrix}
\log y_1(s^{t-1}) \\
\log y_2(s^{t-1})
\end{bmatrix} + \varepsilon(s^t).
\] (9)

where \( \varepsilon(s^t) \sim N(0, \Sigma) \). Notice that I abstract away from the existence of non-tradable goods in this paper and follow a classic literature of international real business cycle models and portfolio choice with only tradable goods (Backus et al., 1992, Heathcote and Perri, 2013).

**Household utility** Each household is risk averse and demands a basket of two tradable goods. Utility functions are assumed to be symmetric across countries. Flow utility has a constant relative risk aversion \( \gamma \) with respect to the aggregate consumption basket \( c^1 \). Final consumption good is aggregated with a CES technology where elasticity of substitution is \( \sigma \), a weight on home good is \( \omega_H \) (home bias), and a weight on foreign good is given as
\( \omega_F = 1 - \omega_H \) as in the complete markets model (section 2).

\[
u(c^1(s^t)) = \frac{c^1(s^t)^{1-\gamma}}{1-\gamma}, \quad c^1(s^t) = \left( \omega_H^\frac{1}{1-\gamma} (c^1_1(s^t))^\frac{\sigma-1}{\sigma} + \omega_F^\frac{1}{1-\gamma} (c^1_2(s^t))^\frac{\sigma-1}{\sigma} \right)^{\frac{\sigma}{\sigma-1}} \tag{10}
\]

where \( c^j_i \) denotes the consumption of good \( i \) by consumers in country \( j \).

**Household budget constraint**  Households decide on the amount of consumption for two tradable goods \((c^1_1, c^1_2)\), and financial portfolio comprised of domestic bonds \((a^1_1)\), foreign bonds \((a^2_1)\), and domestic equity \((\theta^1)\). Their income is generated by returns on financial assets, including dividend payments on equity holdings. Equity holdings \( \theta^1 \in [0, 1] \) give a shareholder claims to a fraction of output \( y^1 \), which is a dividend to the Lucas Tree in country 1. Bond \( i \) promises a unit of final good in country \( i \) in the next period, regardless of the state. To summarize, the budget constraint of household in country 1 is given as the following.

\[
\sum_{i=1}^{2} p^1_i(s^t)c^1_i(s^t) + q_1(s^t)a^1_1(s^t) + e(s^t)q_2(s^t)a^1_2(s^t) + p^1_1(s^t)Q_1(s^t)\theta^1(s^t) \\
\leq a^1_1(s^{t-1}) + e(s^t)a^1_2(s^{t-1}) + p^1_1(s^t) \left( y^1(s^t) + Q_1(s^t) \right) \theta^1(s^{t-1}) \tag{11}
\]

where \( p^1_i \) is relative price of tradable good \( i \) to the final good in country 1, \( q_i \) is the price of bond \( i \), \( e \) is the exchange rate (price of country 2 final good in terms of country 1 final good), and \( Q_i \) is the price of equity \( i \). Notice that the price of final good in country 1 is used as numeraire.

**Stochastic collateral constraints**  It is assumed that the bond purchases are subject to stochastic collateral constraints. The maximum amount of total borrowing by country \( j \) households cannot exceed a fraction \( \chi(s^t) \) of total equity value \( p^j_1(s^t)Q_j(s^t)\theta^j(s^{t-1}) \). Formally, for the household in country 1, collateral constraint for the total borrowing is:

\[
a^1_1(s^t)\mathbb{I}_{(a^1_1(s^t)<0)} + e(s^t)a^1_2(s^t)\mathbb{I}_{(a^2_1(s^t)<0)} \geq -\chi(s^t)p^1_1(s^t)Q_1(s^t)\theta^1(s^{t-1}) \tag{12}
\]
This is under the assumption that in international financial markets, borrowers can default on bonds and the liquidation value of the Tree as a collateral can be zero with a probability \((1 - \chi(s^t))\), following Jermann and Quadrini (2012). More details of the microfoundation behind the collateral constraint can be found in Appendix C.

I assume that the fraction of collateralization \(\chi(s^t)\) follows a Markov process. In particular, there are states where \(\chi(s^t)\) is low enough so that the borrowing constraint is binding, and with a certain probability the constraint is loosened.

**Household’s problem** Each household maximizes her expected utility at time 0 under the budget constraint and borrowing constraint.

\[
\max_{c_1^1(s^t),c_2^1(s^t),a_1^1(s^t),a_2^1(s^t),\theta^1(s^t)} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) u(c_1^1(s^t))
\]

\[
s.t. \sum_{i=1}^{2} p_1^i(s^t)c_1^i(s^t) + q_1(s^t)a_1^1(s^t) + e(s^t)q_2(s^t)a_2^1(s^t) + p_1^i(s^t)Q_1(s^1)\theta^1(s^t) \leq a_1^1(s^{t-1}) + e(s^t)a_2^1(s^{t-1}) + p_1^i(s^t)\left(y_1(s^t) + Q_1(s^t)\right)\theta^1(s^{t-1})
\]

\[
a_1^1(s^t)\mathbb{1}_{(a_1^1(s^t)<0)} + e(s^t)a_2^1(s^t)\mathbb{1}_{(a_2^1(s^t)<0)} \geq -\chi(s^t)p_1^1(s^t)Q_1(s^1)\theta^1(s^{t-1})
\]

**Market clearing** Goods markets clear for each state, and bonds have zero net supply. Equities are owned by domestic households only.

\[
\sum_{j=1}^{2} c_j^1(s^t) = y_1(s^t), \sum_{j=1}^{2} a_j^1(s^t) = 0, \theta^i(s^t) = 1, \forall s^t \in S^t, i = 1, 2
\]

**Net wealth fraction and recursive formulation** In order to solve the model, I transform the household’s problem in a recursive form. I first define wealth fraction \((w)\), which is the country 1 household’s net bond repayment plus dividend payment normalized by the world output. This normalization is designed in a way that in equilibrium, \(w\) is equal to the country 1’s fraction of cash-in-hand (output plus net bond repayment) normalized by the
world output. Formally, net wealth fraction of household $i \in [0, 1]$ at the node $s^{t+1}$ is:

$$w^i(s^{t+1}) = \frac{p^1_1(s^{t+1})y_1(s^t)\theta_1(s^t) + a^1_1(s^t) + e(s^{t+1})a^1_2(s^t)}{p^1_1(s^{t+1})y_1(s^{t+1}) + e(s^{t+1})p^2_2(s^{t+1})y_2(s^{t+1})}$$

(16)

Notice that portfolio decisions at the end of time period $t$ ($a^1_i(s^t), \theta^1(s^t)$) determines an individual’s relative net wealth at the beginning of period $t+1$ ($w^i(s^{t+1})$), depending on the realization of output and hence equilibrium exchange rate in time $t+1$.

In order to express the household’s problem in a recursive way, define aggregate net wealth $W(\cdot)$ which is a sum of individual net wealth fraction of households in country 1:

$$W(\cdot) = \int_{i \in [0,1]} w^i(\cdot) di$$

(17)

Following Kubler and Schmedders (2003), a sufficient statistic for the endogenous states of both countries is $W$, based on zero net supply of bonds and identical individuals. In other words, since the sum of net bond positions in country 1 and 2 should be always zero by the market clearing conditions and the net wealth of all individuals within a country is identical, the aggregate net wealth fraction in country 1 becomes a sufficient statistic for the entire economy and all endogenous variables.

Each individual consumer has rational expectations on the evolution of aggregate net wealth fraction. Conditioning on the previous state $s^t$, a mapping $\Gamma$ from an aggregate net wealth fraction $W(s^t)$ along with a current exogenous state $s_{t+1}$ to another net wealth fraction at $(s^{t+1} = (s^t, s_{t+1}))$ is given as

$$W(s^{t+1}) = \Gamma(W(s^t), s_{t+1}; s^t), \forall s_{t+1} \in S$$

(18)

Notice that consumers form an expectation that maps today’s $W(s^t)$ to tomorrow’s $W(s^{t+1})$ for any pair of states $(s_t, s_{t+1}) \in S \times S$. In equilibrium, given an aggregate net wealth fraction $W(s^t)$ and a policy function $a^1_i(W(s^t), s^t)$, the following equation should be satisfied for any
node $s^{t+1}$.

$$W(s^{t+1}) = \frac{p_1^1 (W(s^{t+1}), s^{t+1}, y_1(s^{t+1}) + a_1^1 (W(s^t), s^t) + e (W(s^{t+1}), s^{t+1}) a_2^1 (W(s^t), s^t))}{p_1^1 (W(s^{t+1}), s^{t+1}, y_1(s^{t+1}) + e (W(s^{t+1}), s^{t+1}) a_2^1 (W(s^t), s^t))}$$

Notice that the market clearing condition for equities ($\theta^i(s^t) = 1$, $\forall s^t \in S^t$) is used for the aggregate law of motion. Also in equilibrium, individual net wealth fraction is equal to the aggregate net wealth fraction, $w^i(s^{t+1}) = W(s^{t+1})$, $\forall i \in [0, 1]$.

A formal definition of consumer’s problem in a recursive form is as follows.

$$V_1 (w(s), \theta^1; W(s), s) = \max_{c^1, c^2} u(c^1, c^2) + \beta \sum_{s'} \pi(s'|s)V_1 (w(s'); W(s'), s')$$

$$s.t. \sum_{i=1}^{2} p_1^i (W(s), s) c_1^i + q_1 (W(s), s) a_1^i + e (W(s), s) q_2 (W(s), s) a_2^i$$

$$\leq w(s) (p_1^1 (W(s), s) y_1(s) + p_2^1 (W(s), s) y_2(s)) + p_1^1 (W(s), s) Q_1 (W(s), s) (\theta^1 - \theta^1')$$

$$a_1^{i'} 1_{(a_1^{i'} < 0)} + e (W(s), s) a_2^{i'} 1_{(a_2^{i'} < 0)} \geq -\chi(s)p_1^1 (W(s), s) Q_1 (W(s), s) \theta^1$$

$$W(s') = \Gamma (W(s), s'; s), \forall s' \in S$$

$$w\left(s', W(s'), a_1^{i'}, \theta^{i'}\right) = \frac{p_1^1 (W(s'), s') y_1(s') \theta^{i'} + a_2^{i'} + e (W(s'), s') a_2^{i'}}{p_1^1 (W(s'), s') y_1(s') + e (W(s'), s') p_2^2 (W(s'), s') y_2(s')}$$

Here, I denote the country’s net wealth fraction as $W$ and individual’s net wealth fraction as $w$, and suppress the history of states $s^t$ into the state of today $s \in S$, exploiting the Markov process of shocks. Accordingly, $s' \in S$ denotes the state of next period and $a_1^{i'}$ is defined as the portfolio choice of today for the payments tomorrow. Consumer’s problem in country 2 is defined analogously, where the country 2’s aggregate net wealth fraction is $1 - W(s)$ due to the zero net supply of bonds. Finally, notice that in equilibrium equity holdings are always 1 ($\theta^1 \equiv 1$), which essentially makes the net wealth fraction $w(\cdot)$ the only endogenous variable.
Recursive competitive equilibrium  Competitive recursive equilibrium is a collection of value functions \( \{V_i(w(s),\theta^1;W(s),s)\}_{i=1,2} \), law of motion for the aggregate net wealth fraction \( \Gamma(W(s),s';s) \), consumption allocation \( \{c^j_i(w(s);W(s),s)\}_{i,j=1,2} \), asset holdings \( \{a^j_i(w(s);W(s),s),\theta^j_i(w(s);W(s),s)\}_{i,j=1,2} \), and prices \( \{p^j_i(W(s),s),Q_i(W(s),s),q_i(W(s),s),e(W(s),s)\}_{i=1,2} \), such that 1) Given the prices and the law of motion for the aggregate net wealth fraction, consumption allocation, asset holdings, and value functions solve each consumer’s problem, and 2) Markets clear.

Numerical algorithm  I provide a global solution of portfolio choice, which implies that equilibrium is known for the time periods with large shocks far from steady state and under the occasionally binding collateral constraints. It is necessary to solve the model globally, especially to address a sudden and large drop of gross capital flows as a result of large negative shocks during the 2008-2009 financial crisis and study the asset holdings in response to the stochastic borrowing limits. In section D of the Appendix, I describe the algorithm in detail.

4 Quantitative Analysis

I calibrate the model to the United States and the Rest of the World (RoW) in years 2000-2019. I first inspect mechanisms of the model. Then, I compare the model predictions of gross capital flows to the data, using the GDP of the US and RoW as an exogenous input of the model simulations. Using the simulated results, I provide a welfare analysis over the sample period and implications of the European Financial Transaction Tax.

4.1 Calibration

One of the key parameters in the calibration is the elasticity of substitution between two tradable goods (\( \sigma \)), or Armington elasticity. In the benchmark model, this elasticity is set to 0.9 following Heathcote and Perri (2002). In the literature, estimations of Armington elasticity range from as low as 0.1 to around 2 for the G-7 countries (Hooper et al., 2000),
Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Targets/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.980</td>
<td>Steady state interest rate 2%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk aversion</td>
<td>1.000</td>
<td>Literature</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Elasticity of substitution</td>
<td>0.900</td>
<td>Heathcote and Perri (2002)</td>
</tr>
<tr>
<td>$\omega_H$</td>
<td>Home bias</td>
<td>0.8</td>
<td>0.5(Imports+Exports)/GDP</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Output persistence</td>
<td>0.928</td>
<td>Estimated from US and RoW GDP</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Output shock std.dev</td>
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<td></td>
</tr>
<tr>
<td>$\rho_{ij}$</td>
<td>Cross-country shock correlation</td>
<td>0.578</td>
<td></td>
</tr>
<tr>
<td>$\chi_L$</td>
<td>Initial collateral constraint</td>
<td>0.013</td>
<td>Indirect inference</td>
</tr>
<tr>
<td>$\chi_C$</td>
<td>Crisis collateral constraint</td>
<td>0.017</td>
<td></td>
</tr>
</tbody>
</table>

which is in line with the usual range for the macro literature. For example, Corsetti et al. (2008) estimates the elasticity to be 0.85 and Stockman and Tesar (1995) sets it equal to 1. Analogous to the Complete Markets sections (Section 2), with smaller $\sigma$ households borrow more in domestic bonds. On the other hand, when $\sigma$ is high enough, then households switch to saving in domestic assets and increase the position with $\sigma$. Therefore, it is important to set the elasticity of substitution that is at a similar magnitude of the observed investment positions as in the data for a rigorous quantitative analysis. In the benchmark model, the simulated external portfolios to GDP is on par with the data comparing its levels and magnitude\(^4\) and endogenous portfolio is such that households borrow in domestic bonds. In the following sensitivity analysis (Section 4.4), the benchmark results are compared with both higher and lower levels of Armington elasticity.

Other parameters are mostly estimated from the US and RoW output data. The home bias parameter ($\omega_H$) is calibrated so that the model steady state matches the average of exports and imports to GDP in the US and RoW during the sample period, which is 25%. Parameters for the output series, including persistence and variance-covariance matrix of exogenous shocks, are estimated based on the US and RoW GDP series.\(^5\) I have calibrated

\(^4\)Average external assets and liabilities to GDP $0.5(\text{Assets} + \text{Liabilities})/\text{GDP}$ in the US after the financial crisis (2012-2019) is 1.56. Analogous results in the benchmark is 1.07.

\(^5\)All series are in log and linear trends are used. More details of the data construction can be found in Appendix G.
the model based on the US estimate of output process. While the spillover effect of exogenous shocks is assumed to be 0, they are fairly correlated with the estimated correlation coefficient ($\rho_{ij}$) of 0.578.

Finally, collateral constraints are calibrated by minimizing the distance of the simulated gross flows and data time series. More specifically, the stochastic collateral constraint $\chi$ is discretized into three points: $\{\chi_L, \chi_C, \chi_H\}$, which represent the initial level of collateral constraint ($\chi_L$), crisis time ($\chi_C$), and the non-binding level ($\chi_H$), respectively. First, the non-binding level $\chi_H$ is set to 0.0185, which is loose enough so that over the course of simulation this level of constraint is never binding. Then, the remaining two constraints ($\chi_L, \chi_C$) are searched jointly$^6$ where the distance between simulated gross flows to GDP and the data is minimized.$^7$ Gross flows and GDP data series are from Bureau of Economic Analysis, International Transactions and Gross Domestic Product tables.$^8$ Transition matrix of the collateral constraints are displayed in the Appendix C. The transition matrix is set so that at the initial level ($\chi_L$), the constraint moves to the non-binding level with the highest probability. Once at the non-binding level ($\chi_H$), it stays at the same level mostly. Finally, once at the crisis level ($\chi_C$), there is a 50 percent chance that the constraint next period stays at the crisis level, which reflects that the financial crisis lasted for 2 years. While there is no explicit target for the transition matrix, I find that the equilibrium allocations are mostly invariant to the transition matrix.

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$^6$BOBYQA algorithm was used for the joint search of non-linear global minimization. More details of the numerical algorithm can be found in the Appendix D.

$^7$More specifically, the distance between the simulated gross flows to GDP $\{\overline{g_{sim}}\}$ and the corresponding data $\{\overline{g_{data}}\}$ is calculated as the following:

$$D = \sum_i [(g_{sim,i} - \overline{g_{sim}}) - (g_{data,i} - \overline{g_{data}})^2]$$

where $\overline{g_{sim}}$ and $\overline{g_{data}}$ are averages of simulated series and data, respectively.

$^8$Asset and liability flows are from International Transactions (Table 1.1), lines 19 and 24. Gross Domestic Product is from GDP&Personal Income, Table 1.1.5., line 1. All series are annual, from years 2000 to 2019.
4.2 Equilibrium portfolios and gross flows mechanism

Based on this calibration, I analyze the main mechanism of consumption risk sharing and collateral constraints. I first describe equilibrium bond portfolios under a loose ($\chi_H$) and a tight ($\chi_L$) collateral constraints. Then, I use impulse response functions to explain the dynamics of consumption, asset holdings, and prices when a negative output shock hits the home country.

**Bond policy functions** In the model, bond policy functions hold the key channel that links output risk sharing, collateral constraints, and gross capital flows. One of the natural states to inspect bond policy functions is the *zero-shock state*, where there are zero shocks for all the exogenous output series ($y_i$), conditional on a given level of collateral constraint ($\chi$). In equilibrium, consumers save in foreign bonds and borrow in domestic bonds for all levels of collateral constraint. At the net wealth fraction of $w = 0.5$ and zero-shock state, consumers save and borrow the same amount of bonds simultaneously, holding zero net positions due to the symmetry. This implies that if the two countries start at the symmetric wealth and the zero-shock state, then they remain at the same state until an exogenous shock hits the output or collateral constraints. Henceforth, the symmetric wealth plus zero-shock state is set as the natural starting point for the following impulse response functions as well as the simulations in the next subsection.

In Figure 4, policy functions of household 1 for both domestic (dashed lines) and foreign (solid lines) bonds are illustrated, conditional on the loose (left) and tight (right) collateral constraints. Two extreme values of collateral constraints, tight ($\chi_L$) and loose ($\chi_H$) levels as calibrated in the previous section, are selected because the crisis level ($\chi_C$) lies in between these two values. Notice that *negative* values of domestic asset ($-a_1'$) are plotted in Figure 4, so that they are directly mapped to the amount of liabilities. In other words, households

---

9This is a similar concept to the Risky Steady State proposed by Coeurdacier et al. (2011) under the symmetric setting and conditional on a certain level of collateral constraint. I avoid using the term risky steady state because collateral constraints do not have a steady state in this model.
Figure 4: Asset and liability position policy functions of country 1 at zero-shock state, loose (left) and tight (right) collateral constraints

borrow in domestic bonds for any level of net wealth or collateral constraint, and this equilibrium portfolio allows a direct mapping of external liabilities to the absolute amount of domestic bond holdings.

Under the loose constraint, households increase the amount of savings in foreign bonds (assets) and reduce borrowings in domestic bonds (liabilities) as the net wealth fraction $w$ increases. As a result, they increase their net savings with higher relative wealth. On the other hand, under the tight constraint, the amount of borrowings by relatively poorer households ($w < 0.5$) are limited to a smaller fraction of their equity holdings, which results in a kink of liabilities and assets at the symmetric level of wealth ($w = 0.5$). When households are relatively poor, they borrow to the maximum amount under the collateral constraint, while as they turn relatively rich, they reduce the amount of liabilities compared to the assets in foreign bonds. The net savings increase with the wealth under tight collateral constraint as well, but the gross positions are bounded by either home or foreign country’s collateral constraints.$^{10}$

$^{10}$Under the loose constraint, there are kinks observed for values of net wealth fraction further from the center. These are the points where the liabilities of either home or foreign country hit the collateral constraints, but not binding over sample simulation path during the calibration.
Dynamic responses of the consumption, bond portfolios, and prices can be best inspected through impulse response functions. Figure 5 describes responses of endogenous variables to a negative shock to the country 1 tradable output \( y_1 \) in time 1, starting from zero-shocks symmetric wealth in time 0 under a loose collateral constraint \( \chi_H \). All other exogenous shocks are set to be zero. In order to highlight the core mechanisms, the impulse response functions described below are calculated without cross-country correlations in output \( \rho_{ij} = 0 \).

As Figure 5 shows, with a decrease in output, domestic goods become more valuable than foreign goods in country 1, which leads to a cheaper basket of foreign goods compared to the domestic basket and hence a decline of real exchange rates (panel (e)). These movements in the real exchange rates result in a lower net wealth fraction for country 1 consumers (panel (d)), because the value of savings in foreign bonds decline while the value of borrowings in domestic bonds increase. Both households reduce their consumption of good 1 by a similar magnitude (panel (b)), as a response to the negative shock in output. Households in country
I also cut down on their bond holdings in absolute values for both domestic and foreign bonds (panel (c)), which reduces their exposures to the changes in net wealth due to exchange rates. If the households do not scale down the amount of assets in foreign bonds and liabilities in domestic bonds, their net wealth would further deteriorate because of a decline in exchange rates. In order to prevent excessive declines in the net wealth, households reduce the size of their external balance sheet in response to a decrease in exchange rates. Afterwards, asset positions recover faster than liabilities, which leads to an eventual savings in the later periods that result in an increase of the net wealth fraction beyond its initial level. Finally, equity value (panel (f)) decreases simultaneously with the lower output, which implies that the borrowing limit as a fraction of equity value tightens on the impact of negative output shock. Despite an appreciation of domestic goods price \(p_1^d\), the equity price \(Q_1\) drop dominates in its magnitude, and the value of collateral decreases as a result. This response in the equity value survives under the tight constraint as well, which exacerbates the tightening of collateral constraints even further when the level of constraint \(\chi\) lowers and output is hit by a negative shock simultaneously.

As the impulse response functions show, households decrease their holdings of both domestic and foreign bonds on the impact of negative output shock. These adjustments result in negative gross capital flows, which account for the changes in both asset and liability positions. The key mechanism is the changes in net wealth through the endogenous movements in exchange rates and the bond positions. When foreign and domestic goods are complements, it is optimal for the households to reduce their consumption on both foreign and domestic goods when the domestic output is hit by a negative shock. Therefore, knowing that exchange rates would decline when such negative shock hits the domestic output, households in equilibrium save in foreign bonds and borrow in domestic bonds. Moreover, they adjust the amount of bond holdings dynamically in order to optimally control the changes in their net wealth in response to the movements of exchange rates and output shocks.
4.3 US Great Recession

In order to test the quantitative performance of the model and analyze welfare implications, I simulate the model for the years of 2000-2019 including the Great Recession in the United States. Real GDP series of the United States and the Rest of the World are taken as exogenous output states \((y_i)\), as plotted in Figure 11 in Appendix, and used as an input of the model simulation\(^{11}\). The model is initiated at the “zero-shock state” as described in the previous section, where collateral constraints begin at the tightest value \((\chi_L)\) in the initial year of 2000 and reaches to the loose constraint \((\chi_H)\) in 2007, following a quadratic function for the years in between. Then, in years 2008-2009, the collateral constraints switch to the crisis level \((\chi_C)\) and from 2010 and on the collateral constraints return to the loose level \((\chi_H)\).

The main exercise is first to generate the equilibrium portfolios on domestic and foreign bonds as an endogenous response to the exogenous output shocks and hence compare gross flows in the model and the data. Then, quantitative importance of two channels, risk sharing and collateral constraint, is analyzed by comparing the benchmark simulations to the counterfactual ones where each channel is shut down. Finally, welfare implications especially during the financial crisis is highlighted as part of the main results. These welfare implications are revisited in the following Sensitivity Analysis section (Section 4.4) with different levels of Armington elasticity.

Gross capital flows In the model, asset and liability flows are calculated as the today’s bond purchases net of the previous period’s. Since households under the benchmark calibration save in foreign bonds and borrow in domestic bonds, asset and liability flows are derived from foreign and domestic bond positions, respectively.\(^{12}\) Gross flows are equal

\[^{11}\text{All real GDP series are in logs and detrended using HP-filter. For more details on the composition of the Rest of the World and data source, see the Appendix.}\]

\[^{12}\text{Asset and liability flows for country 1 are:}\]

\[
\text{asset \_ flows}^1(s^t) = e(s^t)[q_2(s^t)a_2^1(s^t) - a_2^1(s^{t-1})]
\]

\[
\text{liab \_ flows}^1(s^t) = -[q_1(s^t)a_1^1(s^t) - a_1^1(s^{t-1})]
\]
to the sum of asset and liability flows, and are endogenously generated from the model as
the households optimally adjust their bond holdings according to their net wealth fractions,
output shocks, and collateral constraints. The data series for the comparison is sourced
from the Bureau of Economic Analysis, International Transactions data (see Section 4.1:
Calibration for more details).

The model simulated gross flows closely follow the data observations, as shown in Figure
6 with the comparison of the model (solid) and data (dashed) series, both demeaned.13
In particular, magnitudes and key characteristics of simulated gross flows match closely to
the data. Standard deviation of gross flows to GDP in the model is 6.3%, which explains
nearly 90 percent of the data standard deviation (7%). The model also generates the boom-
Figure 7: Gross capital flows, benchmark vs. counterfactuals

Note: Both model simulated and data gross capital flows are de-meaned over the sample period.

bust patterns observed in the data, including a similar magnitude of peak-to-trough collapse in gross flows during the Great Recession. Moreover, asset and liability flows are highly correlated to each other in both model (99%) and data (94%), based on Pearson’s correlation coefficient. Analogous figures for the asset and liability flows can be found in the Appendix H, Figure 12. In the above benchmark model, two channels are at play in generating the simulated gross flows: risk sharing and financial frictions. In the following, quantitative importance of each channel is analyzed by shutting down parts of the channels and comparing the simulated results.

Decomposition: Risk sharing vs. Financial frictions  In order to decompose the movements of gross flows into the risk sharing and financial friction channels, two counterfactual simulations are performed in this section. I first shut down the financial frictions throughout the entire sample periods, and secondly I lift the collateral constraints since 2007 and on. On the left panel of Figure 7, the first counterfactual exercise (dark blue dots) where collateral constraints are fixed at a loose level ($\chi = \chi_H$) for the entire period is compared with the benchmark model simulation (orange solid). This exercise demonstrates that around 45% of the collapse in gross flows during the Great Recession was due to the risk sharing channel,
where the rest is caused by the loosening and tightening of the collateral constraints.

As the second counterfactual exercise, the collateral constraint stays at the loose level \( (\chi = \chi_H) \) since 2007 and on, which implies that there is no crisis-level collateral constraint binding during the Great Recession. On the right panel of Figure 7, the simulated gross flows without the crisis level collateral constraint (dark blue dots) are compared to the benchmark gross flows (orange solid). In year 2008, this counterfactual simulation shows that nearly half of the collapse in gross flows were due to the tightening of the collateral constraint to the crisis level \( (\chi = \chi_C) \). If the collateral constraints remained loose, everything else equal, there would have been a smaller decline in the gross flows and subsequently less pronounced rebound in the aftermath of the financial crisis.

**Welfare analysis** Using the calibrated model, I assess benefits as well as costs of the international investment positions. Households in the United States gained from their asset positions in the run-up to the Great Recession, while losing their consumption since the onset of the financial crisis until 2018. Left panel of Figure 8 depicts the aggregate consumption gains (positive) and losses (negative) of the US (solid green) against the Rest of the World (orange dashed) based on the benchmark model, compared to the financial autarky. It is assumed that under the financial autarky, asset positions for both countries are fixed at zero.
for all states and only goods trading is allowed. The financial autarky model is simulated in the same way as the benchmark model and the endogenous levels of aggregate consumption are used for the welfare analysis.

In 2011, the US aggregate consumption decreased up to 0.14%, which is mirrored by a simultaneous increase in the RoW consumption by the same amount. In other words, there was a significant wealth transfer from the US to RoW in the aftermath of the Great Recession, while the direction of wealth transfer goes the other way around before the crisis. This wealth transfer is summarized as the key endogenous variable $w$, net wealth fraction, as depicted on the right panel of Figure 8 in the solid green line. This net wealth fraction is proportional to the welfare gains and losses in the US compared to the financial autarky. Critically, the movements in the net wealth fraction is due to the endogenous fluctuations in the real exchanges rates, as can be inspected from the model-generated real exchange rates ($e$) on the right panel of Figure 8 in orange dashed line.

In order to further investigate the net gains or losses for the US and RoW households over the sample period, I calculate the present values of consumption for both benchmark and financial autarky models. If the present value of consumption under the benchmark setting is lower than that of financial autarky, consumers did not benefit from the international investment positions despite their welfare gains over part of the years. In units of constant consumption, the US households gained 0.003% of aggregate consumption compared to the financial autarky. On the other hand, households in the Rest of the World lost 0.002% of their consumption on average during the same period. Overall, the total world welfare gained from the open international markets compared to the financial autarky, measured by the total sum of present value of consumption in the US and Rest of the World.
4.4 Sensitivity Analysis

In this section, the benchmark model results of gross flows and welfare analysis are compared to both higher and lower levels of Armington elasticity. As the previous section of complete markets (Section 2) and calibration (Section 4.1) mentioned, Armington elasticity of two tradable goods plays a critical role in determining external portfolios and hence movements of gross flows and international risk sharing. With a lower level of Armington elasticity (0.7), the levels of bond holdings are higher, and the corresponding movements of gross flows are larger in magnitudes compared to the benchmark. As described in Appendix Figure 13, gross flows in 2009 collapse to nearly -30% of GDP, compared to a slightly over -10% of GDP in the benchmark model. The welfare gains and losses for the US during the sample period are amplified under the lower level of Armington elasticity (left panel of Figure 9, orange dashed line), which are calculated in an analogous fashion to the previous section by subtracting the financial autarky aggregate consumption from the benchmark model with a lower elasticity. This amplification is due to the fact that households now hold larger amounts of both external

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\[ I \text{ measure it in a conventional way. For a given consumption stream } \{c_1(s_t)\}, I \text{ find a constant consumption level } \bar{c}_1 \text{ such that } u(\bar{c}_1)(1 - \beta^{19})/(1 - \beta) = \sum_{t=19}^{19} \beta^t u(c_1(s_t)). \]

\[ 15 \text{ While all other parameters remain the same, collateral constraints change with the elasticity because the global minimization as explained in the Calibration section (4.1) applies to each set of parameters. Full description of the collateral constraint parameters can be found in Appendix. } \]
assets and liabilities, and a 1 percent fluctuations in exchange rates bring larger transfers in wealth compared to the benchmark elasticity.

On the other hand, with a higher level of Armington elasticity (1.1), the external asset and liability holdings switch their directions compared to the benchmark elasticity. Specifically, since foreign and domestic tradable goods are substitutes under the Armington elasticity higher than 1, households save (long position) in domestic bonds and borrow (short position) in foreign bonds. This is because when a negative shock hits the domestic output, for example, households now want to substitute their home good consumption with the foreign one by importing more. In order to do so, they save in domestic bonds whose value goes up during bad times due to real appreciations. Due to this switch in external bond positions, the wealth transfer and hence welfare gains (losses) during the sample period mirror the benchmark results. As Figure 9 on the right panel shows, RoW now transfers wealth to the US under a higher elasticity during the Great Recession. While the welfare gains and losses change their direction under a higher elasticity, the magnitude of gross flows and welfare gains/losses remain relatively similar to the benchmark model. However, with a even higher elasticity than this example case of 1.1, the magnitudes of both gross flows and welfare gains (losses) increase, similarly to the lower elasticity case of 0.7.

4.5 European Financial Transaction Tax (FTT)

In September 2011, the European Commission proposed a financial transaction tax that applies to financial transactions of which at least one party is located in any of 27 European Union member states. It levies 0.1% tax on securities trading, and 0.01% on derivative contracts. According to the press release by the European Commission in 2011\textsuperscript{16}, the purpose of the Financial Transaction Tax (FTT) is to collect revenues from the financial sectors who got massive bailouts funded by taxpayers. They expect €57 billion of revenue every year. Currently, implementation of the FTT is on hold but it is considered as an option to finance

\textsuperscript{16}“Financial Transaction Tax: Making the financial sector pay its fair share”
the government expenditures related to COVID-19\textsuperscript{17}. However, it is still far from a EU-wide introduction of FTT because it is hard to reach an unanimous agreement on the benefits of the transaction tax. In the following, I argue that the proposed financial transaction tax will lower the international financial transactions close to zero, resulting in little revenues and hurting international diversification.

**FTT in the baseline model** In the baseline model, I apply 0.1% tax on the transactions of foreign bonds in order to evaluate the effects of FTT. Formally, the consumer’s budget constraint is now as follows.

\[
\sum_{i=1}^{2} p_i^1(s^t)c_i^1(s^t) + q_1(s^t)a_1^1(s^t) + \left[ 1 + \tau \cdot \text{sgn} \left( a_2^1(s^t) \right) \right] e(s^t)q_2(s^t)a_2^1(s^t) + p_1^1(s^t)Q_1(s^t)\theta^1(s^t) \\
\leq a_1^1(s^{t-1}) + e(s^t)a_2^1(s^{t-1}) + p_1^1(s^t) \left( y_1(s^t) + Q_1(s^t) \right) \theta^1(s^{t-1}) + T_1(s^t) \quad (25)
\]

where \( \tau \) is the financial transaction tax, which is set to 0.1% for quantitative analysis. \( \text{sgn}(\cdot) \) is a sign function, so that \( \tau \) works as a tax rate on the absolute value of of \( a_2^1(s^t) \). Tax revenues are equally distributed among the domestic consumers in the form of lump sum transfer, \( T_1(s^t) \). Government runs a balanced budget every period, which satisfies the following equation.

\[
T_1(s^t) = \tau \cdot \text{sgn} \left( a_2^1(s^t) \right) e(s^t)q_2(s^t)a_2^1(s^t)
\]

Therefore, in equilibrium, taxes only distort the price of foreign bonds, not the net wealth of consumers. Country 2 imposes an analogous transaction tax on the transactions of foreign bonds, making it a bilateral taxation as the FTT proposal suggests in 2012\textsuperscript{18}.

In solving the model with financial transaction taxes, all the other model parameters are kept the same as in the benchmark model, Table 1. Compared to the bond positions without the transaction tax, which is 107% of GDP for both domestic short and foreign long positions, after imposing the FTT bond portfolios are essentially zero at the zero-shock state for all

\textsuperscript{17}``The EU’s Tobin Tax Is Being Resurrected” (Laurent, 2020, Bloomberg Opinion)

\textsuperscript{18}``Commission proposes green light for enhanced cooperation on financial transactions tax”
collateral constraints. Since the gross positions are zero, there will be no revenues from the financial transaction taxes to be redistributed. Considering that countries lose the benefits of consumption risk hedging, FTT results in welfare losses for both countries compared to the economy without transaction taxes.

Other studies also have suggested that the benefits from financial transaction taxes are dubious. Pomeranets (2012) empirically shows that the trading volume decreases with the transaction taxes, and the volatility may increase as well. She also casts doubt on the projected revenue collection by the European Commission, considering the effect of substitution and migration. My analysis is in line with her empirical evidence and adds the dimension of international diversification, which supports the argument that benefits of FTT is elusive.

In conclusion, Financial Transaction Tax proposed by the European Commission (EC) may result in low revenues as the financial transaction volumes decrease in response to the tax. A reduction of financial transactions volume implies that countries cannot benefit from the international consumption risk sharing via gross flows, which requires large transaction volumes. Other ways of financial regulations, such as capital ratio regulations, or taxation of other sorts might be more effective in achieving the intended goal of the EC without compromising the benefits of international diversification.

5 Conclusion

This paper shows that international capital flows transfer significant amount of wealth across countries, and hence play a critical role in cross-border risk sharing. In this paper, the main hypothesis is that households share tradable output risks across the border by holding international assets and liabilities, subject to financial frictions. These risk sharing motive and financial frictions are the two main channels that generate the movement of international portfolio positions, and hence gross capital flows. By building a quantitative model, this paper shows that the gross flows benefited the US during the Great Recession, while the severe
financial frictions in 2009 hampered such risk sharing. This paper successfully generates endogenous gross flows from the model, which enables quantitative analysis of capital flows on welfare and decompose the importance of risk sharing and financial friction channels. Admittedly, this paper builds a stylized model of two countries and two bonds, in order to tractably solve the system using a global solution method. Potential expansions of this paper, such as the inclusion of equity portfolio and asymmetric financial frictions, come at the cost of reduced tractability and more computation power, and are left to the future research.

References


Appendices

A Proofs of complete markets: risk sharing intuition

Rewriting the budget constraint for the household 1, for any states \((s, s') \in \{s_1, s_2\}\),

\[
\sum_{i=1}^{2} p_i^1(s') c_i^1(s') + \begin{bmatrix} q_1(s') & e(s') q_2(s') \end{bmatrix} \begin{bmatrix} a_1'(s') \\ a_2'(s') \end{bmatrix} = p_1^1(s') y_1(s') + \begin{bmatrix} 1 & e(s') \end{bmatrix} \begin{bmatrix} a_1(s) \\ a_2(s) \end{bmatrix}
\]

(26)

Simplifying notations:

\[
A(s) \equiv \begin{bmatrix} 1 & e(s) \end{bmatrix}
\]

(27)

\[
B(s) \equiv \sum_{i=1}^{2} p_i^1(s)c_i^1(s) - p_1^1(s)y_1(s)
\]

(28)

\[
C(s) \equiv \begin{bmatrix} q_1(s) & e(s) q_2(s) \end{bmatrix}
\]

(29)

\[
X(s) \equiv \begin{bmatrix} a_1(s) & a_2(s) \end{bmatrix}'
\]

(30)

Then, \(2 \text{ (today’s states)} \times 2 \text{ (tomorrow’s states)} = 4\) budget constraints are:

\[
A(s')X(s) = B(s') + C(s')X(s'), \ \forall s', s \in \{s_1, s_2\}
\]

(31)

The probability of each state is given as half, independent of time.

\[
\pi(s_1) = \pi(s_2) = 0.5
\]

(32)
Conjecture that $X(s_1) = X(s_2) = X^*$. Then, above equation is

$$A(s_j)X^* = B(s_j) + C(s_j)X^* \tag{33}$$

Define a matrix of coefficients for all states.

$$A = \begin{bmatrix} A(s_1) \\ A(s_2) \end{bmatrix}, \quad B = \begin{bmatrix} B(s_1) \\ B(s_2) \end{bmatrix}, \quad C = \begin{bmatrix} C(s_1) \\ C(s_2) \end{bmatrix} \tag{34}$$

$$AX^* = B + CX^* \tag{35}$$

Then, if $(A - C)$ is invertible, we can solve for $X^*$.

$$X^* = (A - C)^{-1}B \tag{36}$$

Now we use the social planner’s allocations to characterize this bond position. First order necessary condition of social planner equalizes marginal utility of tradable $i$ consumption between two countries, $\forall i = 1, 2$.

$$u_1^i(c_1^i(s)) = u_2^i(c_2^i(s)) \tag{37}$$

where $u_j^i(c^j(s))$ is a marginal utility of country $j$ at state $s$ w.r.t. good $i$. More explicitly,

$$u_1^1(c_1^1(s)) = c_1^1(s)^{-\frac{\gamma+1}{\sigma}}(\omega_H/c_1^1(s))^{1/\sigma} \tag{38}$$

$$u_1^2(c_2^1(s)) = c_2^1(s)^{-\frac{\gamma+1}{\sigma}}(\omega_F/c_2^1(s))^{1/\sigma} \tag{39}$$

Given that utility functions are symmetric across countries, social planner’s FOC reduces to
the following equations.

\[
\left( \frac{c^1(s)}{c^2(s)} \right)^{-\sigma\gamma+1} = \frac{c^1_1(s)/\omega_H}{c^2_2(s)/\omega_F} = \frac{c^1_2(s)/\omega_F}{c^2_2(s)/\omega_H} \tag{40}
\]

Moreover, due to symmetry, \( u^1_1(s_1) = u^2_2(s_2) \). Then, plugging in the symmetry to the bond prices, define \( q(H) \equiv q_1(s_1) = q_2(s_2) \) and \( q(L) \equiv q_1(s_2) = q_2(s_1) \). Also, \( e(H) \equiv e(s_1) = \frac{p^*_1(s_1)/p^*_2(s_1)}{p^*_1(s_2)/p^*_2(s_2)} = 1/e(s_2) \). Rewriting the matrix,

\[
A - C = \begin{bmatrix}
1 - q_1(s_1) & e(s_1)(1 - q_2(s_1)) \\
1 - q_1(s_2) & e(s_2)(1 - q_2(s_2))
\end{bmatrix}
\tag{41}
\]

\[
= \begin{bmatrix}
1 - q(H) & (1 - q(L))e(H) \\
1 - q(L) & (1 - q(H))/e(H)
\end{bmatrix}
\tag{42}
\]

\[
det(A - C) = (1 - q(H))^2/e(H) - (1 - q(L))^2e(H) \tag{43}
\]

If \( det(A - C) \) is not 0, then \( A - C \) is invertible.

Define net export of country 1 in state \( s_1 \) as \( nx(H) \)

\[
nx(H) = - \sum_{i=1}^{2} p^*_i(s_1)c^*_i(s_1) + p^*_1(s_1)y_H \tag{44}
\]
Then, asset positions are:

\[ X^* = (A - C)^{-1}B \]

\[ = \frac{1}{\text{det}(A - C)} \begin{bmatrix} 1 - q(H) & - (1 - q(L))e(H) \\ -(1 - q(L)) & (1 - q(H))/e(H) \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{2} p_i^{1*}(s_1)c_i^{1*}(s_1) - p_i^{1*}(s_1)y_H \\ \sum_{i=1}^{2} p_i^{1*}(s_2)c_i^{1*}(s_2) - p_i^{1*}(s_2)y_L \end{bmatrix} \]

(45)

\[ = \frac{1}{(1 - q(H))^2/e(H) - (1 - q(L))^2/e(H)} \begin{bmatrix} 1 - q(H) & - (1 - q(L))e(H) \\ -(1 - q(L)) & (1 - q(H))/e(H) \end{bmatrix} \begin{bmatrix} -nx(H) \\ nx(H) \end{bmatrix} \]

(46)

\[ = \frac{nx(H)}{1 - q(H) - (1 - q(L))e(H)} \begin{bmatrix} -e(H) \\ 1 \end{bmatrix} \]

(47)

The denominator \( 1 - q(H) - (1 - q(L))e(H) \) is excess returns of bond 1 to bond 2 in state \( s_1 \), in terms of country 1 final goods. The equation shows that if the net exports are bigger, and excess returns are smaller, the larger absolute value of positions.

### B Parameters for complete markets

Parameters for the complete markets model section (Section 2) are set to be equal to the values of quantitative analysis part (Section 4), except for the output levels which are arbitrary.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
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</tr>
<tr>
<td>( \gamma )</td>
<td>Risk aversion</td>
<td>1.000</td>
</tr>
<tr>
<td>( \omega_H )</td>
<td>Home bias</td>
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</tr>
<tr>
<td>( y_H )</td>
<td>High output level</td>
<td>1.2</td>
</tr>
<tr>
<td>( y_L )</td>
<td>Low output level</td>
<td>0.8</td>
</tr>
</tbody>
</table>
C Collateral constraints and default renegotiation

Before the beginning of each period, which is prior to the realization of new shocks and repayment of bonds, borrowers decide whether to default or not. If a person defaults, then creditors have rights to seize the Lucas Tree \( i \) that borrower in country \( i \) owns \((\theta^i(s))\) for the current state \( s \). Assume that at the time of liquidation, the market value of Tree is zero with some probability, in the spirit of Jermann and Quadrini (2012).

With probability \( \chi(s) \), the value of Tree \( i \) is \( p^i_1(s)Q^i_1(s) \) when liquidated (in units of country \( i \) final goods), where \( i \) is the country of borrower. With probability \( 1 - \chi(s) \), the Tree does not have any market value. Both parties enter renegotiation process in case of a default, and I assume that borrower has all the renegotiation power. In the following, I derive a collateral constraint where borrower is indifferent from defaulting on the bond and keeping the promise. Without loss of generality, I describe the case of a borrower in country 1.

First, surplus of renegotiation on the borrower’s side when the value of Tree is \( p^i_1(s)Q^i_1(s) \):

\[
\sum_{s'} \pi(s'|s)V_1(w(s'); s', W(s')) - a^1_1(s)\mathbb{I}_{(a^1_1(s)<0)} - e(s)a^1_2(s)\mathbb{I}_{(a^1_2(s)<0)} - p^1_1(s)\theta^1_1(s) \tag{49}
\]

In case that the Tree has zero market value, renegotiation surplus is:

\[
\sum_{s'} \pi(s'|s)V_1(w(s'); s', W(s')) - a^1_1(s)\mathbb{I}_{(a^1_1(s)<0)} - e(s)a^1_2(s)\mathbb{I}_{(a^1_2(s)<0)} \tag{50}
\]

Then, in expectation, renegotiation value for borrower is:

\[
\sum_{s'} \pi(s'|s)V_1(w(s'); s', W(s')) - a^1_1(s)\mathbb{I}_{(a^1_1(s)<0)} - e(s)a^1_2(s)\mathbb{I}_{(a^1_2(s)<0)} - \chi(s)p^1_1(s)\theta^1_1(s) \tag{51}
\]

For the borrower to be indifferent between defaulting and repaying the debt, value of non
defaulting should be at least as big as expected renegotiation value.

\[
\sum_{s'} \pi(s'|s)V_1(w(s'); s', W(s')) \geq \\
\sum_{s'} \pi(s'|s)V_1(w(s'); s', W(s')) - a_1^1(s)\mathbb{1}_{(a_1^1(s)<0)} - e(s)a_2^1(s)\mathbb{1}_{(a_2^1(s)<0)} - \chi(s)p_1^1(s)Q_1(s)\theta^1(s)
\] (52)

Above inequality is equal to the collateral constraint after rearrangement.

D Numerical algorithm

In order to solve the model globally, I use the time iteration algorithm by Kubler and Schmedders (2003), which has been applied to other international portfolio choice models such as Stepanchuk and Tsyrennikov (2015) and Dou and Verdelhan (2015). The algorithm finds equilibrium policy functions starting from an initial guess, by solving a system of first order necessary conditions and Kuhn-Tucker conditions and updating guesses over iterations.

This equilibrium is ‘\(\varepsilon\)-equilibrium’, meaning that the policy functions are solved up to some given critical value \(\varepsilon > 0\) accuracy. Specifically, denote a set of endogenous variables at iteration \(k\) as \(\Omega(k) = \{w(k), c_i^j(k), a_i^j(k), e(k), p_i^j(k), q_i(k), Q_i(k), \xi^j(k)\}, i, j = 1, 2\). Here, I have used the equilibrium condition that aggregate endogenous variables are same as individual ones. Also, \(\xi^j\) is a Garcia-Zangwill parameter that solves for Kuhn-Tucker conditions in country \(j\). Also define all endogenous variables except for net position \(w(k)\) as \(\hat{\Omega}(k) = \Omega(k)/w(k)\), since \(w(k)\) is an endogenous state variable. Functions that are arguments of set \(\hat{\Omega}\) have net wealth fraction and exogenous states as their input \((f : \mathbb{R}^4 \to \mathbb{R}^1, \forall f \in \hat{\Omega})\), which are suppressed in the following expression. Finally, the set of collateral constraints \(\chi = \{\chi_L, \chi_C\}\) that minimizes the distance between simulated series and the data is searched using BOBYQA algorithm (Powell, 2009). The algorithm proceeds as follows.

1. Set up the initial guesses of \(\hat{\Omega}(0)\) and grids for net position \(w\).
I set equispaced grids for net position with 31 points\(^{19}\) for an interval [0.35,0.65], and set steady state prices for \(q_j(0) = \beta\) and \(Q_j(0) = 1/(1 - \beta)\). I start with zero bond positions for all bonds in all countries, \(a^i_j(0) = 0\). Finally, initial Garcia-Zangwill parameters \(\xi_j^i(0)\) are set to \(\sqrt{\chi Q_j(0)}\). Set \(\hat{\Omega}(0) = \hat{\Omega}^o\), where \(\hat{\Omega}^o\) is a set of old policy functions that is updated in every iteration. Exogenous state variables are discretized to 3 points for each output shock using the method by Tauchen (1986), and critical value is set to \(\varepsilon = 10^{-4}\). The initial guess for the collateral constraints \(\chi_0 = \chi_o\) is set, together with the initial loose constraint \(\chi_{H,0} = \chi_{H,o}\).

2. Start the time iteration, given the old guess for the collateral constraints \(\chi_o = \{\chi_{L,o}, \chi_{C,o}\}\).

For any iteration \(k \geq 1\), given the previous iteration’s guess as future endogenous policy functions and prices\(^{20}\), solve a system of first-order conditions and Kuhn Tucker conditions at each grid point of state. I solve the system of equations at precision of \(10^{-5}\), using modified Powell’s non linear solver\(^{21}\). Using the solutions in iteration \(k\), update functions \(f^o \in \hat{\Omega}^o\) as a convex combination of \(f(k) \in \hat{\Omega}(k)\) and \(f^o\): \(f^o = \delta f(k) + (1 - \delta) f^o\).

Algorithm stops when maximum absolute difference of policy, price, and Garcia-Zangwill parameters between \(k^{th}\) iteration and old function across all state grid is less than critical value \(\varepsilon\), \(\max_{(w,s)} \|f(k) - f^o\| < \varepsilon\), \(\forall f \in \hat{\Omega}\), or \(k > K_{max}\).

3. Calculate the distance of simulated gross flows and data and find the new set of parameters \(\chi_n\).

Given the policy functions from step 2., run BOBYQA algorithm that searches for \(\tilde{\chi}\) that minimizes the distance between the simulated gross flows to GDP \(\{gsim_i\}\) and

---

\(^{19}\)For the sensitivity analysis, 23 grid points

\(^{20}\)Here, I need to find the mapping of today’s net wealth fraction \(w(s_t)\) to the tomorrow’s net wealth fraction \(w(s_{t+1})\) for any future state \(s_{t+1}\) by finding a root in equation 19. Since the solution often lies off of the grid points, I use spline methods to interpolate policy functions across endogenous state grid of \(w(s_{t+1})\). I used B-spline method by Habermann and Kindermann (2007).

\(^{21}\)I use HYBRD1 in Minpack. When there are points that cannot be solved, I impute the solution by linearly interpolating the neighboring points that are accurately solved and iterate until all points are solved by the algorithm.
the corresponding data \( \{gdata_i\} \):

\[
D = \sum_i [(gsim_i - \overline{gsim}) - (gdata_i - \overline{gdata})]^2
\]

where \( \overline{gsim} \) and \( \overline{gdata} \) are averages of simulated series and data, respectively. Set the BOBYQA results as \( \chi_n \).

Set \( \chi_o = \chi_n \). Also, if \( \chi_{H,o} \) is not binding for any of the simulation, then \( \chi_{H,n} = \chi_{H,o} \). Otherwise, update \( \chi_{H,n} \) to a higher level than \( \chi_{H,o} \). Go back to step 2, and repeat \( N_{max} \) times. After \( N_{max} \)th iteration, find the parameters that minimize \( D \) across all iterations.

\section*{E Transition matrix, collateral constraints}

Collateral constraint grid points are \( \chi = \{\chi_L, \chi_H, \chi_C\} \), where \( \chi_H = 0.0185 \) is the non-binding constraint, \( \chi_L = 0.013 \) is the initial constraint, and \( \chi_C \) is the crisis level constraint. Transition matrix \( \Pi_{\chi} = \{\pi_{ij}\} \) is the probability of \( \{\chi' = \chi_j|\chi = \chi_i\} \); it is designed so that households expect a non-binding constraint with a the highest probability in the next period (0.5), while there is a good chance of remaining in the same state (0.4). Once at the non-binding level (\( \chi_H \)), it stays at the same level mostly (0.6). Finally, once at the crisis level (\( \chi_C \)), there is a 50 percent chance that the constraint next period stays at the crisis level, which reflects that the financial crisis lasted for 2 years.

\[
\Pi_{\chi} = \begin{bmatrix}
0.4 & 0.5 & 0.1 \\
0.3 & 0.6 & 0.1 \\
0.1 & 0.4 & 0.5
\end{bmatrix}
\] (53)

\section*{F Sensitivity analysis calibration table}
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Targets/Source</th>
</tr>
</thead>
<tbody>
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<tr>
<td>$\chi_L$</td>
<td>Initial collateral constraint</td>
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<td>} Indirect inference</td>
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<tr>
<td>$\chi_C$</td>
<td>Crisis collateral constraint</td>
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</tr>
<tr>
<td>$\chi_H$</td>
<td>Loose collateral constraint</td>
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<td>Armington elasticity = 0.7</td>
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<td>$\chi_C$</td>
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<tr>
<td>$\chi_H$</td>
<td>Loose collateral constraint</td>
<td>0.060</td>
<td></td>
</tr>
</tbody>
</table>

## G Data: Calibration targets

- Source: OECD Statistics, Gross domestic product (expenditure approach: current prices, domestic currency & volume index), exchange rates (period average, national currency per US dollar)

- Rest of the World (RoW) volume index is constructed using an unbalanced panel of available countries. RoW index is an weighted sum of each country’s GDP volume index. Weight is constructed as a fraction of world GDP in USD in each year. World GDP in USD is a sum of available current national currency GDP over USD exchange rate in each year.


H Additional figures

Figure 10: Liability and asset flows, total and debt flows, US against RoW

Note: Debt flows are calculated as the sum of Direct investment: Debt instruments, Portfolio investment: Debt securities, Other investment, and Reserve assets (only for debt asset flows, not for liabilities). Source: Bureau of Economic Analysis and author’s calculations.
Figure 11: Exogenous shocks (output) of US and RoW

Figure 12: Asset and liability flows, data vs. model

Note: Both model simulated and data capital flows are de-meaned over the sample period.
Figure 13: Gross flows, Armington elasticity of 0.7 (left) and 1.1 (right)

Note: All simulated capital flows are de-meaned over the sample period. Benchmark (solid green) is the benchmark model with the Armington elasticity 0.9.